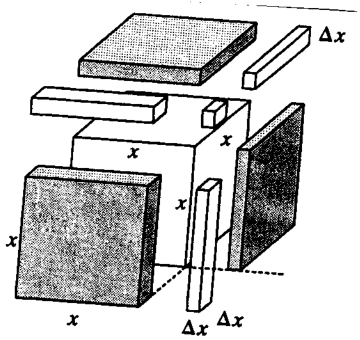
Math 1A Fall 2001: Section 3.3 Solutions

4. (a) At maximum height the velocity of the ball is 0 ft/s. $v(t) = s'(t) = 80 - 32t = 0 \Leftrightarrow 32t = 80 \Leftrightarrow t = \frac{5}{2}$. So the maximum height is $s(\frac{5}{2}) = 80(\frac{5}{2}) - 16(\frac{5}{2})^2 = 200 - 100 = 100$ ft.

(b) $s(t) = 80t - 16t^2 = 96 \Leftrightarrow 16t^2 - 80t + 96 = 0 \Leftrightarrow 16(t^2 - 5t + 6) = 0 \Leftrightarrow 16(t-3)(t-2) = 0$. So the ball has a height of 96 ft on the way up at $t = 2$ and on the way down at $t = 3$. At these times the velocities are $v(2) = 80 - 32(2) = 16$ ft/s and $v(3) = 80 - 32(3) = -16$ ft/s, respectively.

6. (a) $V(x) = x^3 \Rightarrow \frac{dV}{dx} = 3x^2$. $\left.\frac{dV}{dx}\right|_{x=3} = 3(3)^2 = 27\ \text{mm}^3/\text{mm}$ is the rate at which the volume is increasing as x increases past 15 mm.

(b) The surface area is $S(x) = 6x^2$, so $V'(x) = 3x^2 = \frac{1}{2}(6x^2) = \frac{1}{2}S(x)$. The figure suggests that if Δx is small, then the change in the volume of the cube is approximately half of its surface area (the area of 3 of the 6 faces) times Δx. From the figure, $\Delta V = 3x^2(\Delta x) + 3x(\Delta x)^2 + (\Delta x)^3$. If Δx is small, then $\Delta V \approx 3x^2(\Delta x)$ and so $\Delta V/\Delta x \approx 3x^2$.



10. (a) Using $V(r) = \frac{4}{3}\pi r^3$, we find that the average rate of change is:

(i) $\frac{V(8) - V(5)}{8 - 5} = \frac{\frac{4}{3}\pi(512) - \frac{4}{3}\pi(125)}{3} = 172\pi\ \mu\text{m}^3/\mu\text{m}$

(ii) $\frac{V(6) - V(5)}{6 - 5} = \frac{\frac{4}{3}\pi(216) - \frac{4}{3}\pi(125)}{1} = 121.\overline{3}\pi\ \mu\text{m}^3/\mu\text{m}$

(iii) $\frac{V(5.1) - V(5)}{5.1 - 5} = \frac{\frac{4}{3}\pi(5.1)^3 - \frac{4}{3}\pi(5)^3}{0.1} = 102.01\overline{3}\pi\ \mu\text{m}^3/\mu\text{m}$

(b) $V'(r) = 4\pi r^2$, so $V'(5) = 100\pi\ \mu\text{m}^3/\mu\text{m}$.

(c) $V(r) = \frac{4}{3}\pi r^3 \Rightarrow V'(r) = 4\pi r^2 = S(r)$. By analogy with Exercise 7(c), we can say that the change in the volume of the spherical shell, ΔV, is approximately equal to its thickness, Δr, times the surface area of the inner sphere. Thus, $\Delta V \approx 4\pi r^2(\Delta r)$ and so $\Delta V/\Delta r \approx 4\pi r^2$.

12. $V(t) = 5000(1 - \frac{1}{40}t)^2 = 5000(1 - \frac{1}{20}t + \frac{1}{1600}t^2) \Rightarrow V'(t) = 5000(-\frac{1}{20} + \frac{1}{800}t) = -250(1 - \frac{1}{40}t)$

(a) $V'(5) = -250(1 - \frac{5}{40}) = -218.75$ gal/min

(b) $V'(10) = -250(1 - \frac{10}{40}) = -187.5$ gal/min

(c) $V'(20) = -250(1 - \frac{20}{40}) = -125$ gal/min

(d) $V'(40) = -250(1 - \frac{40}{40}) = 0$ gal/min

The water is flowing out the fastest at the beginning — when $t = 0$, $V'(t) = -250$ gal/min. The water is flowing out the slowest at the end — when $t = 40$, $V'(t) = 0$. As the tank empties, the water flows out more slowly.

24. (a) $C(x) = 84 + 0.16x - 0.0006x^2 + 0.000003x^3 \Rightarrow C'(x) = 0.16 - 0.0012x + 0.000009x^2 \Rightarrow C'(100) = 0.13$. This is the rate at which the cost is increasing as the 100th item is produced.

(b) $C(101) - C(100) = 97.13030299 - 97 \approx \$0.13$.

(d) $C''(x) = -0.0012 + 0.000018x = 0 \Rightarrow x = 66\frac{2}{3}$ and $C''(x)$ changes from negative to positive at this value of x. This is where the *marginal cost* changes from decreasing to increasing and so has its minimum value.

16. (a) (i) $\frac{C(6) - C(2)}{6 - 2} = \frac{0.0295 - 0.0570}{4} = -0.006875$ (moles/L)/min

(ii) $\frac{C(4) - C(2)}{4 - 2} = \frac{0.0408 - 0.0570}{2} = -0.008$ (moles/L)/min

(iii) $\frac{C(2) - C(0)}{2 - 0} = \frac{0.0570 - 0.0800}{2} = -0.0115$ (moles/L)/min

(b) Slope $= \frac{\Delta C}{\Delta t} \approx -\frac{0.077}{7.8} \approx -0.01$ (moles/L)/min