Math 1A Fall 2001: Section 4.3 Solutions PART 1

2. (a) g is concave upward on $(-1, 2)$ and $(7, 8)$.

(b) g is concave downward on $(2, 4)$ and $(4, 7)$.

(c) The only point of inflection is $(2, 2)$. Note that 7 is not in the domain of this function.

6. (a) f is increasing on the intervals where $f'(x) > 0$, namely, $(2, 4)$ and $(6, 9)$.

(b) f has a local maximum where it changes from increasing to decreasing, that is, where f' changes from positive to negative (at $x = 4$). Similarly, where f' changes from negative to positive, f has a local minimum (at $x = 2$ and at $x = 6$).

(c) When f' is increasing, its derivative f'' is positive and hence, f is concave upward. This happens on $(1, 3)$, $(5, 7)$, and $(8, 9)$. Similarly, f is concave downward when f' is decreasing — that is, on $(0, 1)$, $(3, 5)$, and $(7, 8)$.

(d) f has inflection points at $x = 1, 3, 5, 7$, and 8, since the direction of concavity changes at each of these values.

10. (a) $f(x) = x/(1+x)^2 \Rightarrow$

$$f'(x) = \frac{(1+x)^2(1) - (x)2(1+x)}{[(1+x)^2]^2} = \frac{(1+x)[(1+x) - 2x]}{(1+x)^4} = \frac{(1+x)(1-x)}{(1+x)^4} = \frac{1-x}{(1+x)^3}.$$ So

$f'(x) > 0 \Leftrightarrow -1 < x < 1$ and $f'(x) < 0 \Leftrightarrow x < -1$ or $x > 1$. So f is increasing on $(-1, 1)$ and f is decreasing on $(-\infty, -1)$ and $(1, \infty)$.

(b) f changes from increasing to decreasing at $x = 1$. $x = -1$ is not in the domain of f. Thus, $f(1) = \frac{1}{4}$ is a local maximum.

(c) $$f''(x) = \frac{(1+x)^3(-1) - (1-x)3(1+x)^2}{[(1+x)^3]^2} = \frac{(1+x)^2[-1(1+x) - 3(1-x)]}{(1+x)^6} = \frac{2x-4}{(1+x)^4}.$$

$f''(x) > 0 \Leftrightarrow x > 2$ and $f''(x) < 0 \Leftrightarrow x < 2$ $(x \neq -1)$. Thus, f is concave upward on $(2, \infty)$ and f is concave downward on $(-\infty, -1)$ and $(-1, 2)$. There is an inflection point at $(2, \frac{2}{9})$.

14. (a) $y = f(x) = x\ln x \Rightarrow f'(x) = x(1/x) + \ln x = 1 + \ln x$. $f'(x) > 0 \Leftrightarrow \ln x + 1 > 0 \Leftrightarrow \ln x > -1 \Leftrightarrow x > e^{-1}$. Therefore f is increasing on $(1/e, \infty)$ and decreasing on $(0, 1/e)$.

(b) f changes from decreasing to increasing at $x = 1/e$, so $f(1/e) = -1/e$ is a local minimum.

(c) $f''(x) = 1/x > 0$ for $x > 0$. So f is concave upward on its entire domain, and has no inflection point.