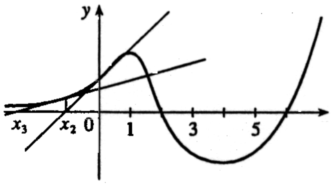
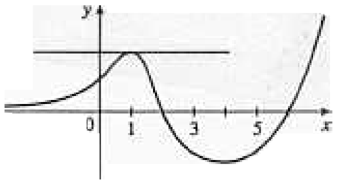
## Math 1A Fall 2001: Section 4.8 Solutions

**4.** (a)



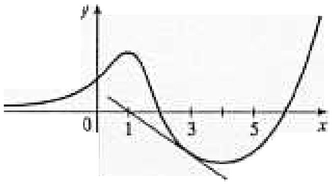

If $x_1 = 0$, then $x_2$ is negative, and $x_3$ is even more negative. The sequence of approximations does not converge, that is, Newton's method fails.

(b)



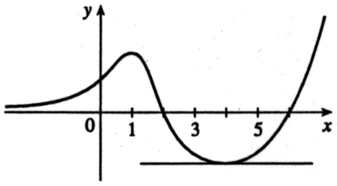

If $x_1 = 1$, the tangent line is horizontal and Newton's method fails.

(c)



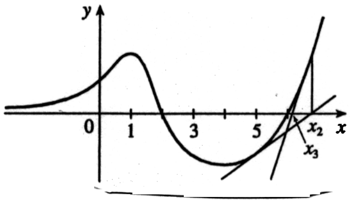

If $x_1 = 3$, then $x_2 = 1$ and we have the same situation as in part (b). Newton's method fails again.

(d)




If $x_1 = 4$, the tangent line is horizontal and Newton's method fails.

(e)

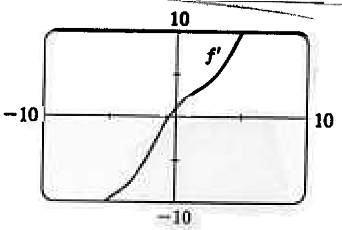



If $x_1 = 5$, then $x_2$ is greater than 6, $x_3$ gets closer to 6, and the sequence of approximations converges to 6. Newton's method succeeds!

**6.** $f(x) = x^3 - x^2 - 1 \;\Rightarrow\; f'(x) = 3x^2 - 2x$, so $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)} = x_n - \dfrac{x_n^3 - x_n^2 - 1}{3x_n^2 - 2x_n}$.

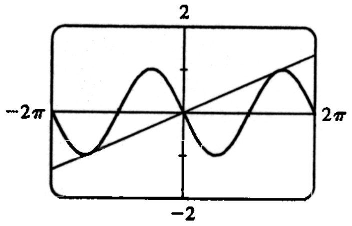
Now $x_1 = 1 \;\Rightarrow\; x_2 = 1 - \dfrac{1-1-1}{3-2} = 2 \;\Rightarrow\; x_3 = 2 - \dfrac{2^3 - 2^2 - 1}{3 \cdot 2^2 - 2 \cdot 2} = 1.625$.

**24.** $f(x) = x^2 + \sin x \;\Rightarrow\; f'(x) = 2x + \cos x$. $f'(x)$ exists for all $x$, so to find the minimum of $f$, we can examine the zeros of $f'$. From the graph of $f'$, we see that a good choice for $x_1$ is $x_1 = -0.5$. Use $g(x) = 2x + \cos x$ and $g'(x) = 2 - \sin x$ to obtain $x_2 \approx -0.450627$, $x_3 \approx -0.450184 \approx x_4$. Since $f''(x) = 2 - \sin x > 0$ for all $x$, $f(-0.450184) \approx -0.232466$ is the absolute minimum.




**10.** $f(x) = x^4 + x - 4 \;\Rightarrow\; f'(x) = 4x^3 + 1 \;\Rightarrow\; x_{n+1} = x_n - \dfrac{x_n^4 + x_n - 4}{4x_n^3 + 1}$. $x_1 = 1.5 \;\Rightarrow\; x_2 \approx 1.323276$, $x_3 \approx 1.285346$, $x_4 \approx 1.283784$, $x_5 \approx 1.283782 \approx x_6$. So the root is 1.283782, to six decimal places.

**26.**




$f(x) = -\sin x \;\Rightarrow\; f'(x) = -\cos x$. At $x = a$, the slope of the tangent line is $f'(a) = -\cos a$. The line through the origin and $(a, f(a))$ is $y = \dfrac{-\sin a - 0}{a - 0}x$. If this line is to be tangent to $f$ at $x = a$, then its slope must equal $f'(a)$. Thus, $\dfrac{-\sin a}{a} = -\cos a \;\Rightarrow\; \tan a = a$.

To solve this equation using Newton's method, let $g(x) = \tan x - x$, $g'(x) = \sec^2 x - 1$, and $x_{n+1} = x_n - \dfrac{\tan x_n - x_n}{\sec^2 x_n - 1}$ with $x_1 = 4.5$ (estimated from the figure). $x_2 \approx 4.493614$, $x_3 \approx 4.493410$, $x_4 \approx 4.493409 \approx x_5$. Thus, the slope of the line that has the largest slope is $f'(x_5) \approx 0.217234$.

**28.** Let the radius of the circle be $r$. Using $s = r\theta$, we have $5 = r\theta$ and so $r = 5/\theta$. From the Law of Cosines we get $4^2 = r^2 + r^2 - 2 \cdot r \cdot r \cdot \cos\theta \;\Leftrightarrow\; 16 = 2r^2(1 - \cos\theta) = 2(5/\theta)^2(1 - \cos\theta)$.

Multiplying by $\theta^2$ gives $16\theta^2 = 50(1 - \cos\theta)$, so we take $f(\theta) = 16\theta^2 + 50\cos\theta - 50$ and $f'(\theta) = 32\theta - 50\sin\theta$. The formula for Newton's method is $\theta_{n+1} = \theta_n - \dfrac{16\theta_n^2 + 50\cos\theta_n - 50}{32\theta_n - 50\sin\theta_n}$. From the graph of $f$, we can use $\theta_1 = 2.2$, giving us $\theta_2 \approx 2.2662$, $\theta_3 \approx 2.2622 \approx \theta_4$. So correct to four decimal places, the angle is 2.2622 radians $\approx 130°$.