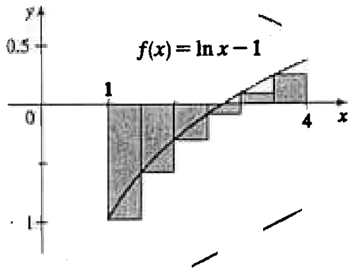
Math 1A Fall 2001: Section 5.2 Solutions

2. $L_6 = \sum_{i=1}^{6} f(x_{i-1})\Delta x$ [. . . .]

$= 0.5\,[f(1) + f(1.5) + f(2)$
$\quad + f(2.5) + f(3) + f(3.5)]$ $\quad [f(x) = \ln x - 1]$
$\approx 0.5(-1 - 0.5945349 - 0.3068528$
$\quad - 0.0837093 + 0.0986123 + 0.2527630)$
$= 0.5(-1.6337217) \approx -0.816861$

The Riemann sum represents the sum of the areas of the two rectangles above the x-axis minus the sum of the areas of the four rectangles below the x-axis; that is, the *net area* of the rectangles with respect to the x-axis.



6. (a) Using the right endpoints to approximate $\int_{-3}^{3} g(x)\,dx$, we have

$$\sum_{i=1}^{6} g(x_i)\,\Delta x = 1[g(-2) + g(-1) + g(0) + g(1) + g(2) + g(3)]$$
$$\approx 1 - 0.5 - 1.5 - 1.5 - 0.5 + 2.5 = -0.5$$

(b) Using the left endpoints to approximate $\int_{-3}^{3} g(x)\,dx$, we have

$$\sum_{i=1}^{6} g(x_{i-1})\,\Delta x = 1[g(-3) + g(-2) + g(-1) + g(0) + g(1) + g(2)]$$
$$\approx 2 + 1 - 0.5 - 1.5 - 1.5 - 0.5 = -1$$

(c) Using the midpoint of each subinterval to approximate $\int_{-3}^{3} g(x)\,dx$, we have

$$\sum_{i=1}^{6} g(\overline{x}_i)\,\Delta x = 1[g(-2.5) + g(-1.5) + g(-0.5) + g(0.5) + g(1.5) + g(2.5)]$$
$$\approx 1.5 + 0 - 1 - 1.75 - 1 + 0.5 = -1.75$$

18. On $[1, 5]$, $\lim_{n\to\infty} \sum^{n} \frac{e^{x_i}}{\ldots}$ ... $= 5/n$ and $x_i = 5i/n]$

$$\lim_{n\to\infty} \frac{5}{n}\left[\sum_{i=1}^{n} 1 + \frac{250}{n^3}\sum_{i=1}^{n} i^3\right]$$

$$\ldots \frac{1250}{n^4} \cdot \frac{n^2(n+1)^2}{4}\Bigg]$$

$$= \lim_{n\to\infty}\left[5 + 312.5 \cdot \frac{(n+1)^2}{n^2}\right] = \lim_{n\to\infty}\left[5 + 312.5\left(1 + \frac{1}{n}\right)^2\right]$$

$$= 5 + 312.5 = 317.5$$

30. (a) $\int_0^2 g(x)\,dx = \frac{1}{2} \cdot 4 \cdot 2 = 4$ (area of a triangle)

(b) $\int_2^6 g(x)\,dx = -\frac{1}{2}\pi(2)^2 = -2\pi$ (negative of the area of a semicircle)

(c) $\int_6^7 g(x)\,dx = \frac{1}{2} \cdot 1 \cdot 1 = \frac{1}{2}$ (area of a triangle)

$\int_0^7 g(x)\,dx = \int_0^2 g(x)\,dx + \int_2^6 g(x)\,dx + \int_6^7 g(x)\,dx = 4 - 2\pi + \frac{1}{2} = 4.5 - 2\pi$

32. $\int_{-2}^{2} \sqrt{4 - x^2}\,dx$ can be interpreted as the area under the graph of $f(x) = \sqrt{4 - x^2}$ between $x = -2$ and $x = 2$. This is equal to half the area of the circle with radius 2, so $\int_{-2}^{2} \sqrt{4 - x^2}\,dx = \frac{1}{2}\pi \cdot 2^2 = 2\pi$.

36. $\int_0^3 |3x - 5|\,dx$ can be interpreted as the area under the graph of the function $f(x) = |3x - 5|$ between $x = 0$ and $x = 3$. This is equal to the sum of the areas of the two triangles, so $\int_0^3 |3x - 5|\,dx = \frac{1}{2} \cdot \frac{5}{3} \cdot 5 + \frac{1}{2}(3 - \frac{5}{3})4 = \frac{41}{6}$.

40. $\int_2^{10} f(x)\,dx - \int_2^{7} f(x)\,dx = \int_2^{7} f(x)\,dx + \int_7^{10} f(x)\,dx - \int_2^{7} f(x)\,dx = \int_7^{10} f(x)\,dx$