Math 1A Fall 1999: Final Exam

1) Find the following limits, if they exist:

(a) $\lim_{x\to 0}\left(\frac{\sin 3x}{4x}\right)$ (b) Find $\lim_{x\to k}\left(\frac{x^3-kx^2}{x^2-k^2}\right)$ (c) Evaluate $\lim_{x\to 0}(1+\sin 2x)^{\frac{3}{x}}$

2) Find the derivative $\frac{dy}{dx}$:

(a) $xy^2+x^2\sin y=1$ (b) $y=\frac{e^{6x}+5x^2+\sqrt{x}}{x}$ (c) $y=\int_0^{\sin x}\sqrt{1-t^3}\,dt$ (d) $y=x^{x\sin x}$

3) Find the indicated integrals:

(a) $\int\frac{x}{\sqrt{x^2+1}}dx$ (b) $\int(4-2\cos\theta)^3\sin\theta\,d\theta$ (c) $\int_5^{10}\frac{x}{\sqrt{x-1}}dx$

4) Use linear approximation or differentials to approximate $\sqrt[4]{15}$. Make use of the fact that $2^4=16$.

5) The surface area of a sphere is given by $S=4\pi r^2$ where r is the radius of the sphere. The diameter of the sphere is 6cm with an error of ± 0.2cm.
Use differentials to estimate the error and the percent error in the calculated surface area.

6) Show that the parabola $y=x^2$ and the line $x+2y-3=0$ intersect at right angles at one of their points of intersection.

7) A farmer wishes to fence a rectangular pasture having a total area of 12,000 square meters. She wants to divide it into two parts with a fence across the middle. Fencing around the outside costs $7.50 per meter, but the farmer can use less expensive fencing at $3 per meter as the divider. What dimensions will result in the least cost?

8) A kite is 150 feet high and is moving horizontally away from a boy at the rate of 20 feet per second. How fast is the string being paid out when the distance from the boy to the kite is 250 feet?

9) Sketch the graph of $y=\frac{x^2-x}{(x+1)^2}$. Plot any intercepts, stationary points, and points of inflection. Show any horizontal and vertical asymptotes.

10) A rapid transit trolley moves with a constant acceleration and covers the distance between two points 300 feet apart in 8 seconds. Its velocity as it passes the second point is 50 feet per second.
(a) What is its acceleration?
(b) What is the velocity of the trolley as it passes the first point?

11) Verify that $f(x)=\frac{4x}{4-x}$ satisfies the hypothesis of the Mean Value Theorem over the interval [1,3] and find all values that satisfy the conclusion of the theorem.

12) (a) Estimate the area under the graph of $y=x^3+2$ between $x=1$ and $x=4$ by a Riemann Sum using three intervals of equal width and using left-hand endpoints.
(b) Estimate the same area using three intervals of equal width and using right-hand endpoints.
(c) Give the Trapezoid Rule estimate for this area, again using three intervals.
(d) Give the exact value for this area.
(e) Calculate the average value of the function $f(x)=x^3+2$ over the interval [1,4].