

5. (12 points) Let $f(x) = \sqrt{x}$.

(i) Find the linearization of $f(x)$ at $x = 4$.

(ii) Use the linearization from part (i) to approximate $\sqrt{4.1}$.

(iii) Explain why the estimate in part (ii) is larger than $\sqrt{4.1}$.

$$\text{i) } L(x) = \frac{1}{4}(x-4) + 2$$

$$\text{ii) } L(4.1) = 2.025$$

$$\text{iii) } \text{SINCE } \frac{d^2}{dx^2} \sqrt{x} = -\frac{1}{4}x^{-3/2} < 0,$$

IS CONCAVE DOWN ON $(0, \infty)$.

THIS MEANS THAT ALL OF ITS TANGENT LINES, INCLUDING $L(x)$, LIE ABOVE IT.