

Math 1A Fall 2001: First Math 1A Exam, Spring 1999

1. (24 points) Differentiate the following functions.

(a) $f(x) = \sqrt{x^3 + 1}$

(b) $f(x) = \frac{\ln x}{x}$

(c) $f(x) = \log_{10}(\cos x)$

(d) $j(x) = x^{\sin x}$

(e) $j(x) = \sin^6(\ln(7x))$

(f) $j(x) = x \sin^{-1} x$

2. (16 points) An aircraft is climbing at a 30° angle to the horizontal. Its speed is a constant 500 miles per hour.

a. How fast is the aircraft gaining altitude?

b. Assuming it's noon and the sun is directly overhead, how fast is the plane's shadow moving along the ground?

3. (18 points) Consider the function $y = y(x)$ defined implicitly by the equation

$$\ln y + \ln(xy) = x.$$

a. Find the derivative $\frac{dy}{dx}$ by implicit differentiation.

b. Now solve the original equation $\ln y + \ln(xy) = x$ for y to express y explicitly as a function of x . Differentiate this expression to find the derivative $\frac{dy}{dx}$ again.

c. Do your answers to the two parts above agree? Explain briefly.

4. (20 points) Consider the function

$$y = x^2 \cdot e^{-\frac{1}{x}}.$$

a. Where is this function increasing? Where is it decreasing?

b. Where is this function concave up? Where is it concave down?

c. For what values of x does this function have a local maximum? A local minimum? A point of inflection?

d. On the axes below, sketch a graph of this function. **Note:** Don't worry about the y -coordinates of the graph; just make sure it agrees with your answers to parts a, b and c.

5. (12 points) For each of the following functions, find all values of x such that $f'(x) = 0$.

(a) $f(x) = \sqrt{x} \ln(x^3), x > 0$

(b) $f(x) = \sin^2 x + \cos x, -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$

6. (10 points) a. Using the limit definition of the derivative, find the derivative $f'(5)$ at $x = 5$ of the function

$$f(x) = \frac{2}{x-3}.$$

b. Use this to find the equation of the tangent line to the graph of f at $x = 5$.