MULTIPLE SOLUTIONS TO SOME WORK PROBLEMS

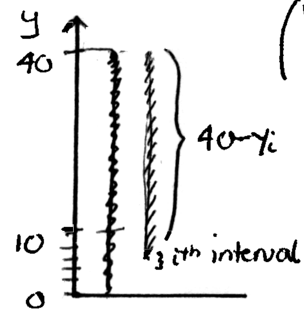
§6.5 # 8, 10, 13

8. Cable's density 60 lbs/40 ft = 1.5 lbs/ft.

VERSION ①

y=0 at bottom of rope

Strategy: Partition the distance [0,10] into n equal pieces



$$\begin{pmatrix}\text{Work done}\\ \text{to move through}\\ \text{the ith interval}\end{pmatrix} = \text{force}_i \cdot \Delta y$$

force = weight of rope $\Rightarrow$ $\text{Work}_i = 1.5\,(40-y_i)\,\Delta y$

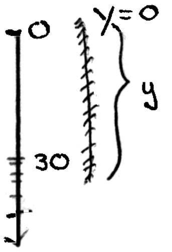
$$\int_0^{10} \underbrace{1.5\,(40-y)}_{\text{force}}\, \underbrace{dy}_{\text{distance}} = 1.5\left[40y-\frac{y^2}{2}\right]_0^{10}$$

$$1.5\,[400-50] = \frac{3}{2}\,350 = 175\cdot 3 = 525 \text{ ft-lbs}$$

Version 2

Same strategy, different arrangement of y-axis.

y=0 at top of rope.



Partition the distance [30,40] into n equal pieces

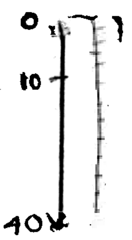
$$\begin{pmatrix}\text{Work done to}\\ \text{move through the}\\ \text{ith interval}\end{pmatrix} = \text{force}_i \cdot \Delta y = 1.5\, y\, \Delta y$$

$$\int_{30}^{40} \underbrace{(1.5\,y)}_{\text{force}}\, \underbrace{dy}_{\text{distance}} = 1.5\frac{y^2}{2}\Big|_{30}^{40} = \frac{3}{2}\left(\frac{1600}{2}-\frac{900}{2}\right)$$

$$= \frac{3}{2}(350) = 525 \text{ ft-lbs}$$

Version 3

y = 0 at the top of rope.



Strategy - chop up the rope; The weight of a slice is $1.5\,\Delta y$.

for $y \in [0,10]$ each bit of rope moves up a different distance

for $y \in [10,40]$ each bit of rope moves 10 ft.

Lets start with the rope from y=10 to y=40. Each bit moves 10 ft. Partition [10,40]

$$\begin{matrix}\text{Work}_i \\ \text{(to move the ith slice of rope)}\end{matrix} = \text{force}_i \cdot \text{distance}_i$$

$$1.5\,\Delta y \cdot 10$$

$$\int_{10}^{40} 1.5\cdot 10\, dy = 15\, y\Big|_{10}^{40} = (15)(30) = 450 \text{ ft-lbs}$$

(You don't need an integral for this (1.5)(30) lbs of rope × 10 ft = 45 lbs · 10 ft = 450 ft-lbs

Now look at the rope from 0 to 10. Partition [0,10].

$$\text{Work}_i = \underline{\text{force}_i} \cdot \underline{\text{distance}_i} = 1.5\,\Delta y \cdot y_i$$

$$\int_0^{10} 1.5\, y\, dy = 1.5\,\frac{y^2}{2} = \frac{3}{2}\cdot 50 = 75 \text{ ft-lbs}$$

Work to move the top 10 ft. of rope + Work to move the rest = 450 + 75 = 525

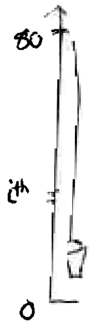
You can do this also with y=0 at the bottom of the rope; Note: if you're not lifting up the whole rope it's easier to partition <u>distance</u> as opposed to the rope itself.

#10. To do this we can 1st consider the work done to lift the empty bucket:

$4\text{lbs} \cdot 80\text{ ft} = 320 \text{ ft-lbs}$

Alternatively, add 4 lbs to the weight of the water.
Here I'll look at the water only and then add 320 ft-lbs.

<u>version 1</u>



$y=0$ is the bottom of the well.
Partition $[0,80]$ into n equal pieces

$\begin{pmatrix}\text{Work done to}\\ \text{move the water}\\ \text{through the } i^{th}\\ \text{interval}\end{pmatrix} = \text{work}_i = \text{force}_i \cdot \Delta y$

force_i = weight of water at a ht, y_i.
The bucket loses .2 lb/sec and the bucket is being raised at 2 ft/sec or $\frac{1}{2}$ sec/ft.

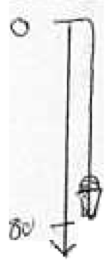
unit analysis can show: $40 - .2\,\frac{\text{lb}}{\text{sec}} \cdot \frac{1}{2}\,\frac{\text{sec}}{\text{ft}} \cdot y_i\,\text{ft} = 40 - .1y_i$ lbs

$\text{Work}_i \approx (40-.1y_i)\,\Delta y$

$\text{Work} = \int_0^{80}(40-.1y)\,dy = 40y - \frac{.1y^2}{2}\Big|_0^{80} = 3200 - \frac{640}{2} = 3200-320$

Now add 320 for the bucket $\Rightarrow$ 3200 ft-lbs

<u>version 2</u>



$y=0$ is the top of the well. Partition $[0,80]$.

$\text{Work}_i = \text{force}_i \cdot \Delta y$

$= 40 - .1(80-y_i)\,\Delta y$

$\text{Work} = \int_0^{80}(40-8+.1y)\,dy = 40y - 8y + .1\frac{y^2}{2}\Big|_0^{80} = 32y + \frac{1}{20}y^2\Big|_0^{80}$

$= 3200 - 320$

Now add 320 $\Rightarrow$ 3200 ft-lbs

<u>version 3.</u> It takes 40 seconds to pull up the bucket
Partition the time interval $[0,40]$

$\begin{pmatrix}\text{work done in}\\ i^{th} \text{ time}\\ \text{interval}\end{pmatrix} = \text{force}_i(\text{distance})_i$

since we're pulling at 2 ft/sec the distance is $2 \cdot \Delta t$

$\text{Force}_i = \text{weight}_i = 40 - .2\,\text{lbs/sec} \cdot t_i\,\text{sec} = 40 - .2t_i$

$\text{work}_i = (40-.2t_i)(2\,\Delta t)$

$\text{Work} = \int_0^{40}(40(2) - .4t)\,dt = 80t - \frac{.4t^2}{2}\Big|_0^{40} = 3200 - 320$

Now add 320 $\Rightarrow$ 3200.