

# Calculus, Series and Differential Equations

## Mathematics 1b Spring 2003

### *Course Information and Syllabus*

#### **Course Content and Goals:**

About four hundred years ago, Galileo wrote “The book of the universe is written in the language of mathematics.” Although the language of mathematics has evolved over time, the statement has as much validity today as it did when it was written. In Mathematics 1b you will become more well-versed in the language of modern mathematics and learn more about its applications to other disciplines. Math 1b is a second semester calculus course for students who have previously been introduced to the basic ideas of differential and integral calculus. Over the semester we will study three (related) topics, topics that form a central part of the language of modern science:

- applications and methods of integration,
- infinite series and the representation of functions by infinite polynomials known as power series,
- differential equations.

The material we take up in this course has applications in physics, chemistry, biology, environmental science, astronomy, economics, and statistics. We want you to leave the course not only with computational ability, but with the ability to use these notions in their natural scientific contexts, and with an appreciation of their mathematical beauty and power.

We will start the semester with integration. You should already be familiar with the definite integral, its definition as the limit of Riemann sums, and its calculation using antiderivatives and u-substitution. The definite integral enables us to tackle a multitude of problems in a wide array of fields; we will use the notion of Riemann sums, (slicing, approximating, and summing) to apply integration in various contexts. More important than any one particular application is the ability to apply the integration as appropriate in problem solving; we will devote time to developing your skill in doing this. We’ll spend a short time at the very beginning of the course looking at methods of integration including the integration analogues of both the Chain Rule and Product Rule for differentiation. We’ll look at some transformations of integrals that enable us to tackle them more efficiently and then move on to applications of the definite integral.

In the second unit of the course we will approximate familiar functions like  $e^x$ ,  $e^{-x^2}$ ,  $\sin x$  and  $\cos(x^2)$  by polynomials. The functions listed are challenging to evaluate and some are challenging to integrate. By using polynomials of increasingly large degree we can get increasingly good approximations to the functions. In fact, we will find that each of these functions has a representation as an infinite polynomial. study infinite sums.

(You already are aware that a rational number such as  $\frac{1}{3}$  can be represented by an infinite sum,  $(\frac{3}{10} + \frac{3}{100} + \frac{3}{1000} + \frac{3}{10000} + \dots)$ , for the case at hand). Actually, irrational numbers such as  $e$  and  $\pi$  have representations as infinite sums as well.)

We will end with differential equations, equations modeling rates of change. Differential equations permeate quantitative analysis throughout the sciences (in physics, chemistry, biology, environmental science, astronomy) and social sciences. In a beautiful and succinct way they provide a wealth of information. By the end of the course you will appreciate the power and usefulness differential equations and you will see how the work we have done with both series and integration comes into play in analyzing their solutions.

**Text:** *Single Variable Calculus: Concepts and Contexts* by James Stewart. Second edition, Brooks/Cole 2001. This text is available at the Harvard Coop. There will be supplementary material available as well.

**Class and Problem Sessions:** Math 1b is taught in sections that meet three hours per week. Each section of Math 1b has a Course Assistant who will be in class, collect and correct homework assignments, and hold weekly problem sessions. You are strongly encouraged to attend these problem sessions as they are an integral part of the course and will be generally be devoted to working problems and amplifying the lecture material. The pace of the course is rather fast, so these sessions should be particularly valuable to you in learning the material. A schedule of all problem sessions will be posted on the course web site; feel free to go to any Math 1b Course Assistant’s Problem Session.

**Homework:** Problems are an integral part of the course; it is virtually impossible to learn the material and to do well in the course without working through the homework problems in a thoughtful manner. Don’t just crank

through computations and write down answers; *think* about the problems posed, the strategy you employ, the meaning of the computations you perform, and the answers you get. It is often in this reflection that the greatest learning takes place.

An assignment will be given at each class meeting. Unless otherwise specified, the assignment is due at the following class meeting and will be returned, graded, at the subsequent class. If you miss a class, then you are responsible for obtaining the assignment and handing it in on time. Solutions put together by the course assistants will be available on the course website. When your homework assignments are returned to you, you can consult the solutions for help with any mistakes you might have made. Problem sets must be turned in on time. When computing your final homework grade, your lowest two homework scores will be dropped if you are in a TTh section and your lowest three homework scores will be dropped if you are in a MWF section.

Note that homework problems will sometimes look a bit different from problems specifically explicitly discussed in class. To do mathematics you need to think about the material, not simply follow recipes. (Following preset recipes is something computers are great at. We want you to be able to do more than this.) Giving you problems different from those done in class is consistent with our goal of teaching you the art of applying ideas of integration and differentiation to different contexts. Feel free to use a calculator or computer to check or investigate problems for homework. However, an answer with the explanation “because my calculator says so” will not receive credit. Use the calculator as a learning tool, not as a crutch. Calculators will not be allowed on examinations due in part to equity issues.

You are welcome to collaborate with other students on solving homework problems; in fact, you are encouraged to do so. However, write-ups you hand in must be your own work, you must be comfortable explaining what you have written, and there must be a written acknowledgement of collaboration with the names of you coworkers.

Odd-numbered problems are solved in the *Student Solution Manual*. After working on the problems on your own, you are free to consult this manual provided you acknowledge the use of this manual in your submitted work.

**Math Help:** The following sources of math help are available without any appointment:

- weekly problem sessions lead by course assistants: go to as many as you like
- office hours held by your section leader
- the Math Question Center: in Loker Common 8-10 pm every night except Friday and Saturday.

**Exams:** Exams are common and given in the evenings. Please keep these exam dates free from conflicts:

Technique Test	Tues. February 18	6-7 pm.	SC C
Exam 1	Tues. March 4	7:30-9:30 pm	SC D
Exam 2	Thurs. April 10	7-9 pm	Yenching Auditorium
Final Exam	Tues. May 20	TBA	TBA

*There will be an optional Technique re-Test available on Thursday, February 27: 7-8 in SC D . The higher of your two scores counts in the computation of your course grade. The first test is not optional.*

Calculators will not be allowed on examinations, due in part to equity issues. We will make sure that problems on the exams require minimal calculation to allow you to spend your time demonstrating your mathematical knowledge as opposed to your calculating ability. We expect you to express your ideas, line of reasoning, and answers clearly. If, for some reason you have an unmovable conflict with an exam time you are expected to e-mail the course head at least one full week in advance so we can make an attempt at accommodation.

### Grading Policy:

Your course grade will be determined as follows:

midterm score: 40% First Exam + 45% Second Exam + 15% Technique Test

Course score: Take the higher of

45% Final Exam + 35% midterm score + 20% homework

30% Final Exam + 50% midterm score + 20% homework

40% Final Exam + 50% midterm score + 10% homework

You must pass the final to earn more than a C in the course.

**Coursehead:** Robin Gottlieb Science Center 429, (617) 495-7882, gottlieb@math.harvard.edu

## TENTATIVE WEEK-BY-WEEK SYLLABUS: January, 2003

### INTEGRATION

February 3 - 7

- Integration by substitution: the Chain Rule in Reverse
- Integration by Parts: the counterpart of the Product Rule.
- Partial Fractions

February 10 - 14

- Recap plus with trig substitution - lightly.
- Definite integral as limit of Riemann Sums. Area (and interpreting area under rate curve), Volumes
- Volumes of revolution

*Presidents' Day: no class on Monday February 17*

February 18- 21

*Technique Test: Tuesday, February 18*

- Density and slicing. Total mass from density; total population from population density.
- Slicing continued. Begin work.

February 24 - 28

*Technique Retest: Thurs. Feb. 27*

- Work: pulling, pushing, and pumping.
- Average value; arc length; begin improper integrals
- Improper integrals.

March 3-7

- Probability

**Exam 1:** *Tuesday, March 4*

### SERIES

- Motivation for studying series. Geometric sums and geometric series.
- Sequences and series. Infinite series: nth term test for divergence and harmonic series.

March 10 - 14

- Introduction to comparison analysis. Direct comparison (to geometric series).
- Comparison analysis continued. Integral test.  $p$ -series. Comparison and limit comparison tests.
- Taylor polynomials: approximating a function by a polynomial.

March 17 - March 21

- Taylor series: representing a function by a power series.
- Absolute and conditional convergence. Alternating Series Test and accompanying error estimate.
- Ratio test.

March 31 - April 4

- Power Series. Getting new power series from old ones by substitution, differentiation and integration.
- Taylor polynomials and the Taylor remainder: Taylor's Inequality. Taylor series and MacLaurin Series.

- More Taylor Series, including the Binomial Series. Applications of Taylor Polynomials.

April 7 - 11

- Series Review and Recap

**Exam 2:** *Thursday, April 10*

**DIFFERENTIAL EQUATIONS**

- Modeling with differential equations.
- Slope fields:  $\frac{dy}{dt} = 1$ ,  $\frac{dy}{dt} = t$ ,  $\frac{dy}{dt} = y$ , and  $\frac{dy}{dt} = -\frac{t}{y}$ . Guess and check solutions.

April 14 - 18

- Checking solutions. Separation of variables.
- Autonomous first order differential equations: qualitative analysis of solutions.
- Solving first order linear differential equations.  
Euler's method summarized: what to do when you can't solve.

April 21 - 25

- Systems of differential equations.
- Continue systems of differential equations.
- Vibrating springs and second order linear homogeneous differential equations with constant coefficients.

April 28 - May 2

- Finish second order linear homogeneous differential equations with constant coefficients.
- Series solutions to differential equations.