Last Name: ______________________

First Name: ______________________

Math 1a
Second Midterm
November 26, 2002

Please circle your section:

Nick Ramsey 9 MWF | Grigor Grigorov 10 MWF | Florian Herzig 10 MWF | John Mackey 11 MWF

Laurent Berger 10 TTh | Ken Chung 10 TTh | Stephanie Yang 10 TTh

Laurent Berger 11:30 TTh | Olga Plamenevskaya 11:30 TTh

Problem	Points	Score
1	14	
2	14	
3	12	
4	14	
5	10	
6	12	
7	12	
8	12	
Total	100	

1. (14 points) Find the indicated derivatives:

(a) $\frac{d}{dx}(x^3 \cdot \sec x \cdot \ln x)$

(b) $\frac{d}{dx}(\sin e^{(x^2)})$

(c) $\frac{d}{dx}(x \log_3 x + \frac{3^x}{x})$

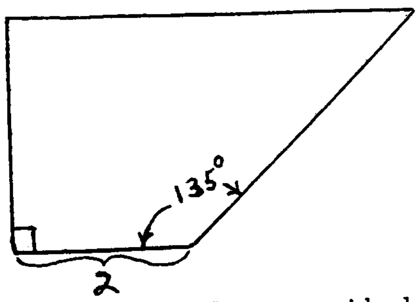
(d) $\frac{d}{dx}(\tan^{-1}(3x^2))$

(e) $\frac{d}{dx}(4(\tan x)^5)$

(f) $\frac{d}{dx}(\cos(x + \cos x))$

(g) $\frac{d}{dx}(2 + \cos x)^x$

2. A trapezoid has a bottom base of length 2 feet and angles to the bottom base fixed at 90° and 135° (see the figure).



(a) (4 points) What is the area of the trapezoid when its height is equal to h feet?

(b) (5 points) If the area decreases at a rate of 1 square foot per minute, at what rate is the height changing when $h = 2$?

(c) (5 points) If the height increases at a rate of 1 foot per minute, at what rate is the area changing when $h = 2$?

3. Let $f(x) = x^3 + 3x^2 - 24x + 8$.

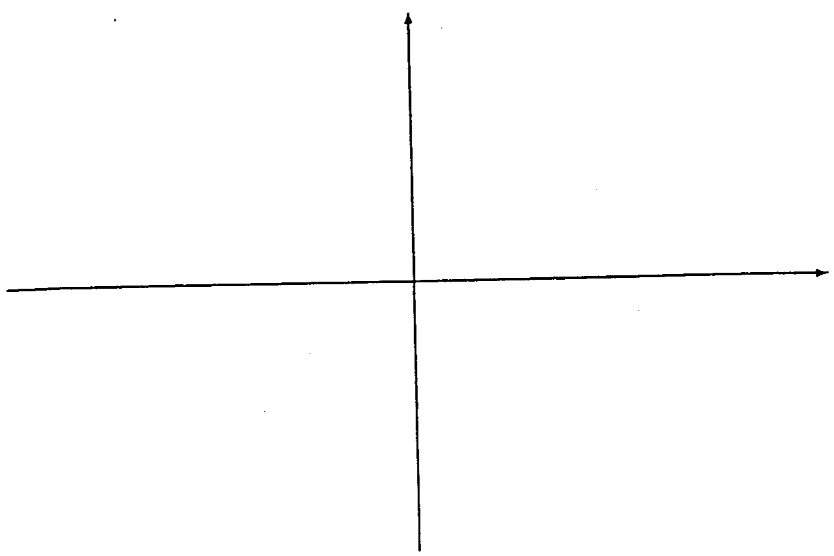
(a) (2 points) Find the intervals on which f is increasing and decreasing.

(b) (2 points) Find the local maxima and local mimima of f (Clearly state if each point is a maximum or a minimum).

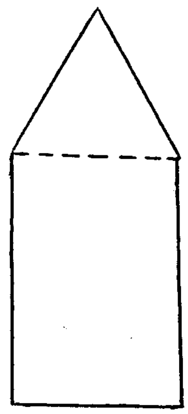
(c) (2 points) Find the intervals on which f is concave up and concave down.

(d) (2 points) Find the inflection point(s) of f.

(e) (4 points) Using the above information, sketch a graph of f.



4. (14 points) A window with the shape of a rectangle capped by an equilateral triangle (see the figure) is to have a perimeter of 100 feet. What is the largest possible area of such a window?



5. (10 points) Find the limits below, if they exist:

(a)

$$\lim_{x \to \pi/2} \frac{\sin x}{x}$$

(b)

$$\lim_{x \to 0} \left(\frac{1}{\sin(2x)} - \frac{1}{2x} \right)$$

(c)

$$\lim_{x \to 0+} (e^x - 1)^{1/\ln x}$$

(d)

$$\lim_{x \to \infty} \left(1 + \frac{1}{3x}\right)^x$$

(e)

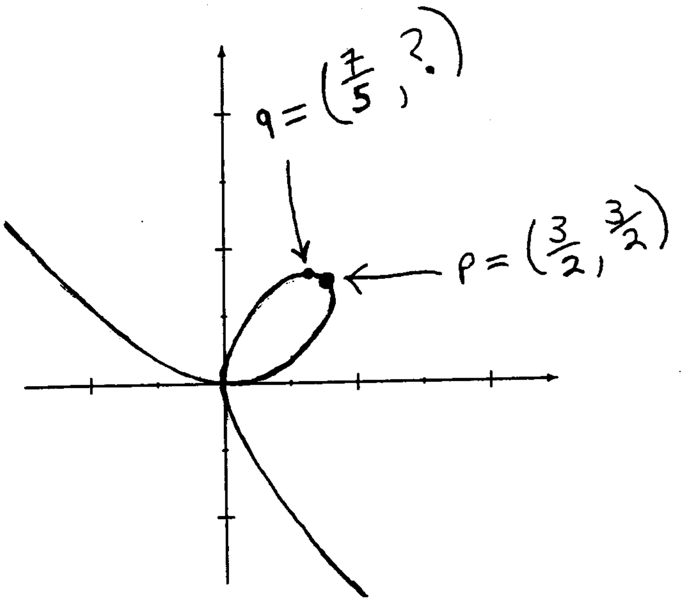
$$\lim_{x \to 0} \frac{\sin^2 x}{\tan^2 x}$$

6. (a) (4 points) State the Mean Value Theorem.

   (b) (4 points) Sketch a function on $[1, 3]$ which is continuous, but doesn't satisfy the conclusion of the Mean Value Theorem.

   (c) (4 points) Given that $f(-2) = 1$ and that $f'(x)$ exists and is less than 3 for all real numbers, show that $f(2) < 13$, using the Mean Value Theorem.

7. Consider the graph of the equation $x^3 + y^3 = 3xy$, as shown in the figure below:



(a) (4 points) Find $\frac{dy}{dx}$ in terms of x and y.

(b) (4 points) Find the equation of the line tangent to the graph at the point $p = (\frac{3}{2}, \frac{3}{2})$.

(c) (4 points) Use the tangent line in part (b) to approximate the y-coordinate of the point labeled q, using the fact that the x-coordinate of q is $\frac{7}{5}$.

8. (12 points) For each of the functions below, the graph is one of the six curves show in the figures at the bottom of this page. In each case, match the function to the graph:

$$f(x) = \frac{\sin x}{x} \qquad g(x) = \frac{1-x}{e^x}$$

$$h(x) = \frac{-\ln x}{x} \qquad k(x) = \frac{\cos x}{x}$$