1a. Evaluate the limit of $\ln(\cos x)/x^2$ as x approaches 0.

$$\lim_{x\to 0} \frac{\ln(\cos x)}{x^2} = \lim_{x\to 0} \frac{\frac{1}{\cos x}\cdot(-\sin x)}{2x} = \lim_{x\to 0} \frac{1}{\cos x}\cdot(-)\frac{\sin x}{2x}$$

$$= \lim_{x\to 0} \frac{1}{\cos x}\cdot(-)\cdot\frac{\cos x}{2} = \boxed{-\frac{1}{2}}$$

1b. Find the derivative of the function $f(x) = \tan^{-1}(2x + 1)$.

$$f'(x) = \frac{1}{(2x+1)^2+1}\cdot 2 = \boxed{\frac{1}{2x^2+2x+1}}$$

1c. Differentiate the function $f(x) = x^{\ln(x)}$. Hint: consider $\ln(f)$.

$$\ln(f) = \ln(x)\cdot\ln(x) = (\ln(x))^2$$

$$\Rightarrow \frac{f'}{f} = 2\ln(x)\cdot\frac{1}{x}$$

$$\Rightarrow f' = 2\ln(x)\cdot\frac{1}{x}\cdot x^{\ln x} = \boxed{2\cdot\ln(x)\cdot x^{\ln x - 1}}$$

2a. Here are four candidates for the definition of the derivative of the function f(x) at the point x=a. Circle the ones which are correct.

$$\lim_{x \to a} \frac{f(x) - f(a)}{x - a} \qquad \lim_{x \to a} \frac{f(x) - f(a)}{x}$$

$$\lim_{h \to 0} \frac{f(x+2h) - f(x)}{2h} \qquad \lim_{x \to a} f'(x) - f(a)$$

b. Find all lines through the origin which are tangent to the curve

$$x^2 - 4x + y^2 + 2 = 0 .$$

Note that the origin is not a point on this curve.

Let the line be $l: ax + b = y$

$(0;0) \in l \Rightarrow 0 \cdot a + b = 0 \Rightarrow b = 0$

$(x-2)^2 + y^2 - 2 = 0 \Rightarrow (x-2)^2 + y^2 = 2$

$(x-2)(-2) + y \cdot 0 = 2$

$\Rightarrow x = 1 \quad y = \pm 1 \Rightarrow a \cdot 1 = 1 \Rightarrow a = 1$

$a \cdot 1 = -1 \quad \text{or } a = -1$

$\Rightarrow$ the lines are $y = x$ and $y = -x$

3. An object travels along a straight line, starting at time t=0, with position given by the formula:

$$s(t) = (2/3)t^3 - 7t^2 + 20t + 8.$$

When the velocity is positive, it is moving to the right.

a. Find a formula for the velocity at time t.

$v = \frac{ds}{dt} = \frac{2}{3} \cdot 3 \cdot t^2 - 7 \cdot 2 \cdot t + 20 = 2t^2 - 14t + 20$

b. Find a formula for the acceleration at time t.

$a = \frac{dv}{dt} = 4t - 14$

c. When is the object moving to the right?

when $v > 0 \Leftrightarrow 2t^2 - 14t + 20 > 0 \Leftrightarrow t^2 - 7t + 10 > 0$
$\Leftrightarrow (t-2)(t-5) > 0 \quad t < 2$ and $t > 5$

d. When is the velocity decreasing?

when $a < 0 \Leftrightarrow 4t - 14 < 0 \Leftrightarrow t < 3.5$

e. When does the object change direction?

- always moving on a straight line
- from part c) $\Rightarrow$ up to $t = 2$ moves right, from 2 to 5 moves left, from ~~5~~ $t = 5$ on moves right again

4. The derivative f'(x) of the function f(x) is graphed below. The numbers indicate the area between the graph of f'(x) and the x-axis. Assume that f(5) = 5.

a) Calculate the value f(0) =

$f(5)-f(0)=\int_0^5 f'(x) = 3-1=2 \Rightarrow f(0)=f(5)-2=5-2=3$

b) Calculate the integral $\int_3^{10} f''(x)\,dx =$ $\underbrace{f'(10)}_{0} - \underbrace{f'(3)}_{0} = 0$

(from the graph)

c) Where does the function f(x) take its maximum value on the interval [0,10] ?

- critical points $f'(x)=0$ at $3, 5, 10$

$f(5)-f(3)=-1 \Rightarrow f(3)=6$

$f(10)-f(5)=6 \Rightarrow f(10)=11 \Rightarrow f(10)=\max$

d) What is the maximum value of f(x) on this interval?

$f(10)=11 = \max$