Last Name: ______________________________

First Name: ______________________________

Math 1a Final Exam: Tuesday May 23, 2000

Instructor: Robert Winters
Course Assistant: Peter Hamel

Question	Points	Score
1	8	
2	8	
3	10	
4	6	
5	10	
6	8	
7	10	
8	10	
9	8	
10	8	
11	8	
12	6	
Total	100	

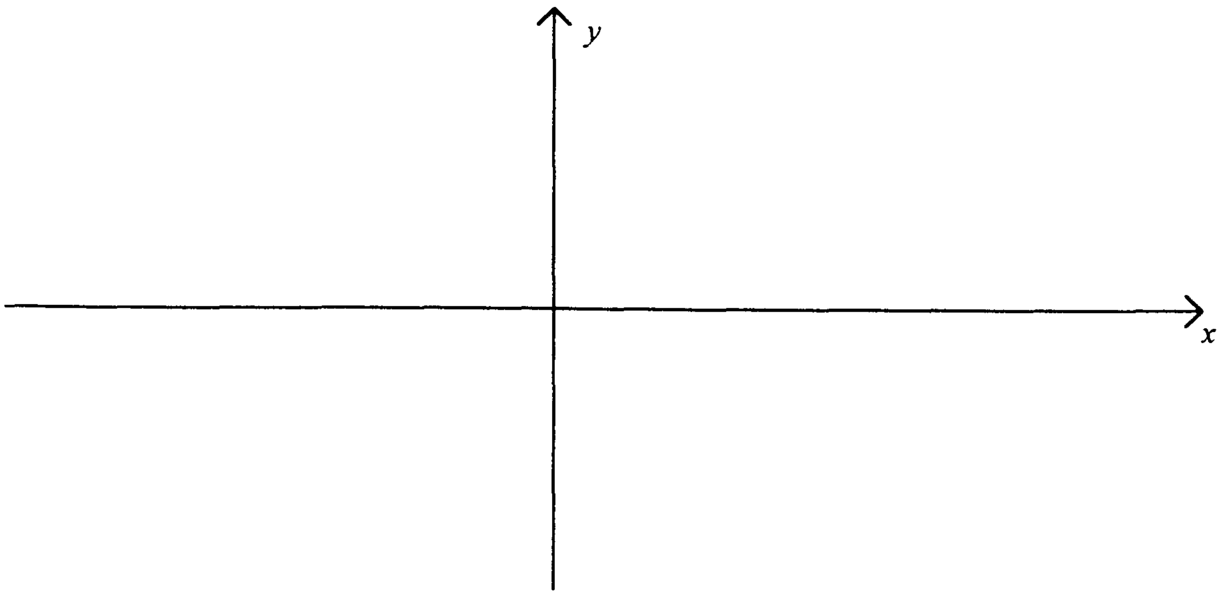
This exam is 3 hours long. No calculators are allowed.

Solutions may be continued on the reverse sides, but you should make clear where any work is to be found. Justify your answers carefully. No partial credit can be given for unsubstantiated answers.

(1) A slippery slope is shaped in the form of the curve with equation $xy^3 = 80$ where x and y are measured in feet. A sled slides down this slope in such a way that its height y is dropping at a rate of 0.8 ft/sec when its height is 10 ft. How fast is its horizontal position x changing at that point?

(2) Show that for a given fixed perimeter the isosceles triangle (two sides of equal length) that gives the most area is an equilateral triangle (three sides of equal length).

(3) (a) Sketch the graph of the function $f(x) = \dfrac{x^2 + 5x + 4}{x}$ showing intercepts, critical points, local maxima and minima, points of inflection, and asymptotes.



(b) Describe in one sentence the general behavior of this graph for very large values of x.

(4) (a) Find the linear approximation for the function $f(x) = \dfrac{1}{1 - e^x \sin 5x}$ for values of x near zero.

(b) Use this approximation to estimate the value of $f(0.03)$.

(5) A vehicle travels along a straight road in such a way that its acceleration (measured in ft/sec^2) is given at any time t by $a(t) = \begin{cases} \frac{t}{10} & \text{for} \quad 0 \le t \le 20 \\ 2 & \text{for} \quad 20 \le t \le 40 \end{cases}$. If the vehicle is initially at rest, answer the following:

(a) What is the vehicle's velocity at $t = 30$?

(b) How far has the vehicle traveled when $t = 30$?

(6) A 40 meter long pole is made of a material where the density (measured in kg/m) at a point x meters from one end is given by $D(t) = \begin{cases} \frac{t}{10} & \text{for} \quad 0 \le t \le 20 \\ 2 & \text{for} \quad 20 \le t \le 40 \end{cases}$.

(a) Find the total mass of the pole.

(b) Find the average density of this pole.

(7) Find the derivative $\frac{dy}{dx}$ in parts (a) - (d):

(a) $y = (\tan^{-1} x)^x$

(b) $y = e^{6x} + 6x^e + 6^x$

(c) $y = \int_{x^3+x}^{100} \sqrt{p^2 - p}\ dp$

(d) $x^3y^3 + xy = 10$ at the point (2,1).

8) Find the indicated integrals:

(a) $\int \frac{4x}{\sqrt{9x^2+25}}\,dx$

(b) $\int \frac{4}{\sqrt{9x^2+25}}\,dx$

(c) $\int_{5}^{21} \frac{x+3}{\sqrt{3x+1}}\, dx$

(d) $\int_{3}^{+\infty} \frac{dx}{(5x+1)^{3/2}}$

(9) The equation $x^3 - 30x^2 + 200x = 0$ can be easily factored to give the three roots $x = 0$, $x = 10$, and $x = 20$. Use this information and one iteration of Newton's method to find an approximate root for the equation $x^3 - 30x^2 + 200x + 1 = 0$ near $x = 10$.

(10) A factory owner today sells 100,000 widgets at a price of $1.00 each. It costs 50 cents to produce each widget, so the current profit is $50000. Market research indicates that for every 10 cent increase in price, the demand will drop by 10000 widgets. What price will yield the greatest profit?

(11) When a bank posts an interest rate of 4% on a savings account, the actual money you earn depends on how frequently this interest is compounded. For example, if P_0 dollars is invested and interest is compounded once each year, after t years this will yield $P(t) = P_0(1+.04)^t$ dollars in the bank. The same amount compounded n times per year will yield $P(t) = P_0(1+\frac{.04}{n})^{nt}$ dollars in the bank after t years. This is equivalent to compounding once per year at a slightly higher interest rate known as the Annual Percentage Rate (APR).

Find the APR in the case where the interest is compounded continuously, i.e. the limiting case where n approaches infinity, if the posted rate is 4%.

(Hint: Find the appropriate limit in the case where $t = 1$ year.)

(12) Verify that $f(x)=\frac{x^2+5x+4}{x}$ satisfies the hypothesis of the Mean Value Theorem over the interval [1,5] and find all values that satisfy the conclusion of the theorem.