1) We are given $xy^3 = 80$
and that $\frac{dy}{dt} = -0.8$ ft/sec when $y = 10$ ft

And we want $\frac{dx}{dt}$ when $y = 10$ ft

Use implicit differentiation:

$$xy^3 = 80$$

$$\Rightarrow \frac{dx}{dt} y^3 + 3y^2 \frac{dy}{dt} \cdot x = 0$$

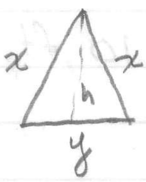
$$\Rightarrow \frac{dx}{dt} = \frac{-3y^2 x \frac{dy}{dt}}{y^3}$$

when $y = 10$, $x = \frac{80}{10^3} = .08$

$$\Rightarrow \frac{dx}{dt} = \frac{-3(.08)(-.8)}{10} = .019 \text{ ft/s}$$

2)

$$2x + y = p \implies x = \frac{p-y}{2}$$



$$\text{Area} = A = \frac{1}{2} b h$$

see that by Pythagoras

$$h = \sqrt{x^2 - \left(\frac{y}{2}\right)^2}$$

$$\implies \text{Area} = \frac{1}{2} y \sqrt{x^2 - \left(\frac{y}{2}\right)^2}$$

now substitute in for x:

$$A(y) = \frac{1}{2} y \sqrt{\left(\frac{p-y}{2}\right)^2 - \left(\frac{y}{2}\right)^2}$$

$$= \frac{1}{2} y \sqrt{\frac{p^2 - 2py + y^2 - y^2}{4}}$$

$$A(y) = \frac{1}{4} y \sqrt{p^2 - 2py}$$

$$A'(y) = \frac{1}{4}\sqrt{p^2 - 2py} + \frac{\frac{1}{4} y}{\sqrt{p^2 - 2py}} \cdot (-2p) \cdot \frac{1}{2}$$

now set this equal to zero and multiply by $4\sqrt{p^2 - 2py}$:

$$\implies p^2 - 2py - py = 0$$

$$\implies p - 3y = 0$$

$$\implies y = \frac{1}{3} p$$

Since the endpoints of the interval give $A = 0$ ft^2 (minima), we see that $y = x = \frac{1}{3}p$ gives the maximal area.

3) $f(x) = \dfrac{x^2 + 5x + 4}{x}$

First, examine limits: $\lim_{x \to \infty} f(x) = +\infty$, $\lim_{x \to -\infty} f(x) = -\infty$
also, we see that we have a vertical asymtote at $x = 0$, and $\lim_{x \to 0^+} = +\infty$, $\lim_{x \to 0^-} = -\infty$

Also, see that $f(x) = \dfrac{(x+4)(x+1)}{x}$

$\Rightarrow f(x) = 0$ for $x = -1, -4$, so we have the only roots at these points.

$$f'(x) = \frac{x(2x+5) - (x^2+5x+4)}{x^2} = \frac{2x^2 + 5x - x^2 - 5x - 4}{x^2}$$

$$= \frac{x^2 - 4}{x^2}$$

And we see $f'(x) = 0$ for $x = \pm 2$

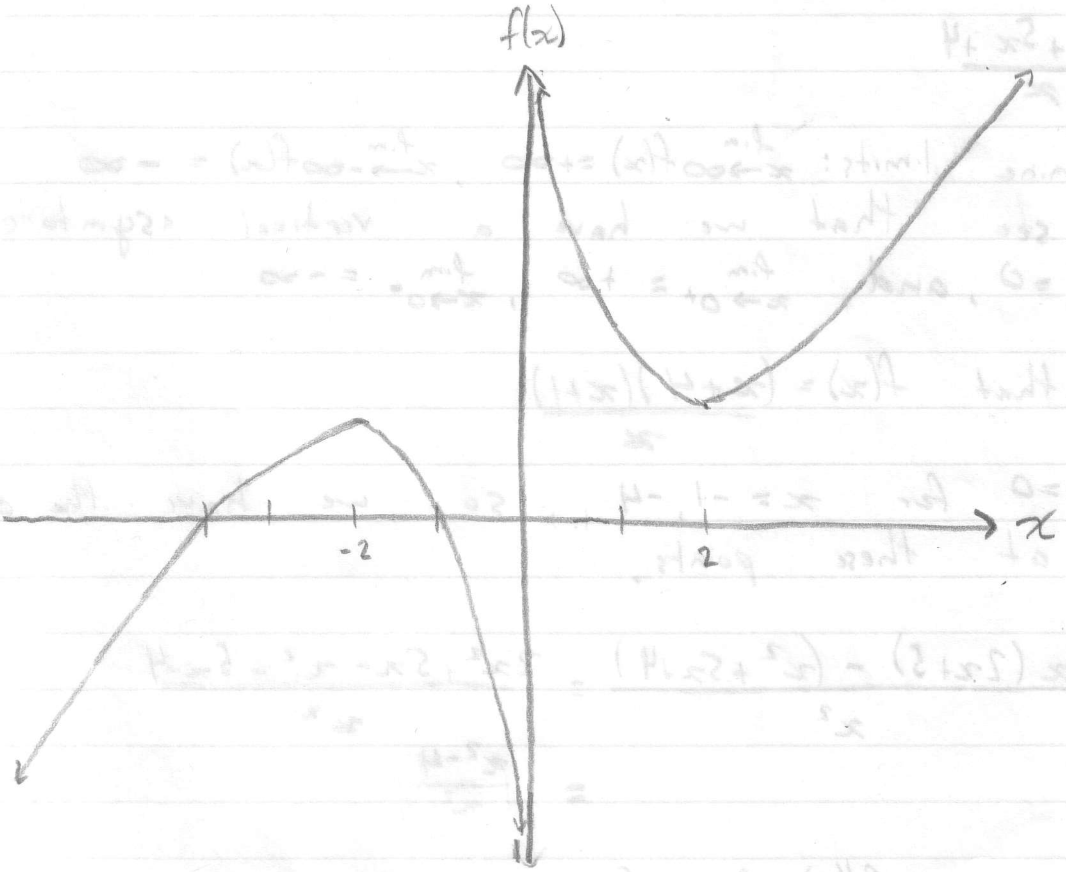
and since $f'(x) = \dfrac{x^2-4}{x^2} = 1 - \dfrac{4}{x^2}$

$$\Rightarrow f''(x) = -2 \cdot (-4) \cdot x^{-3} = \frac{8}{x^3}$$

and so $f''(x) < 0$ for $x < 0$
and $f''(x) > 0$ for $x > 0$

So f is concave down on $(-\infty, 0)$
and f is concave up on $(0, \infty)$

So now we can graph it!



f(x)
x
-2
2