

MATHEMATICS 1A, FINAL EXAM REVIEW QUESTIONS

ALEXANDER ELLIS

A quick disclaimer: Professor Leingang has not told me what will be on the final exam. These questions are only my best guess at what **some** of the test questions will look like. This is in no way a complete review of the course. In particular, you should also review maximization problems and related rates problems.

Other good sources of review material are the two midterms and the practice questions indicated for them, your homework, and the review problems at the end of each chapter of the textbook.

1. Evaluate the following limits.

- (a) $\lim_{x \rightarrow 0} \frac{\cos x - 1}{x}$
- (b) $\lim_{x \rightarrow 2} \frac{x^2 - 5x + 6}{x - 2}$
- (c) $\lim_{x \rightarrow \infty} \left(1 + \frac{\pi}{x}\right)^x$
- (d) $\lim_{x \rightarrow \infty} \frac{\tan^{-1}(x^2)}{x^2}$
- (e) $\lim_{x \rightarrow 1} \frac{x^{1000} - 1}{x - 1}$
- (f) $\lim_{x \rightarrow \infty} \sqrt[5]{x^{1/x}}$

2. For the following functions $f(x)$, evaluate $f'(x)$ and $f''(x)$ using the definition of the derivative.

- (a) $y = \sqrt{x}$
- (b) $y = x^2 + x$

3. Find dy/dx by any means necessary. If possible, express it in terms of x only.

- (a) $y = \sin^{-1}(\ln x)$
- (b) $y^2 = x^3$
- (c) $\tan y = \sin x$
- (d) $y = xe^{x^2}$
- (e) $y = \sin x^2$
- (f) $y = \frac{x^2 \sin x}{\cos x}$

4. For the following functions $f(x)$, find all asymptotes (vertical and horizontal), intervals on which f is increasing and decreasing, intervals on which f is concave up and down,

extrema, points of inflection. Then sketch them on a domain and range that captures all of the features you have found.

(a) $f(x) = \frac{x^2-1}{x+2}$

(b) $f(x) = \frac{x^3-3x^2-4x+12}{x-2}$

(c) $f(x) = \ln(x^2)$

(d) $f(x) = \sin(1/x)$

(e) $f(x) = \tan^{-1}(1/x^2)$

5. Use the Squeeze Theorem to find $\lim_{x \rightarrow \infty} \frac{\cos(\ln x)}{x^4}$.

6. Consider a rectangle where each side is also the hypotenuse of a right isosceles triangle (which is outside the rectangle). If the perimeter of the rectangle is 16, what is the maximum possible area of the entire figure?

7. A ladder of length 13 feet is leaning against a wall on the left. The end on the floor slides to the right as the end on the wall, falls down the wall. At the moment when the base is 5 feet from the wall, the end against the wall is falling at 0.5 feet per second. At what rate is the area of the triangle formed by this arrangement decreasing?