

Last Name: _____

First Name: _____

Second Midterm – MATH 1a
19 October 1996
9:00-10:30 P.M.

Your section (circle one):

Otto Bretscher TTH 10	Otto Bretscher TTh 11.5	Matthew Emerton MWF 10	Matthew Emerton MWF 11
Stavros Garoufalidis	Bob Kaplan	David Pollack	

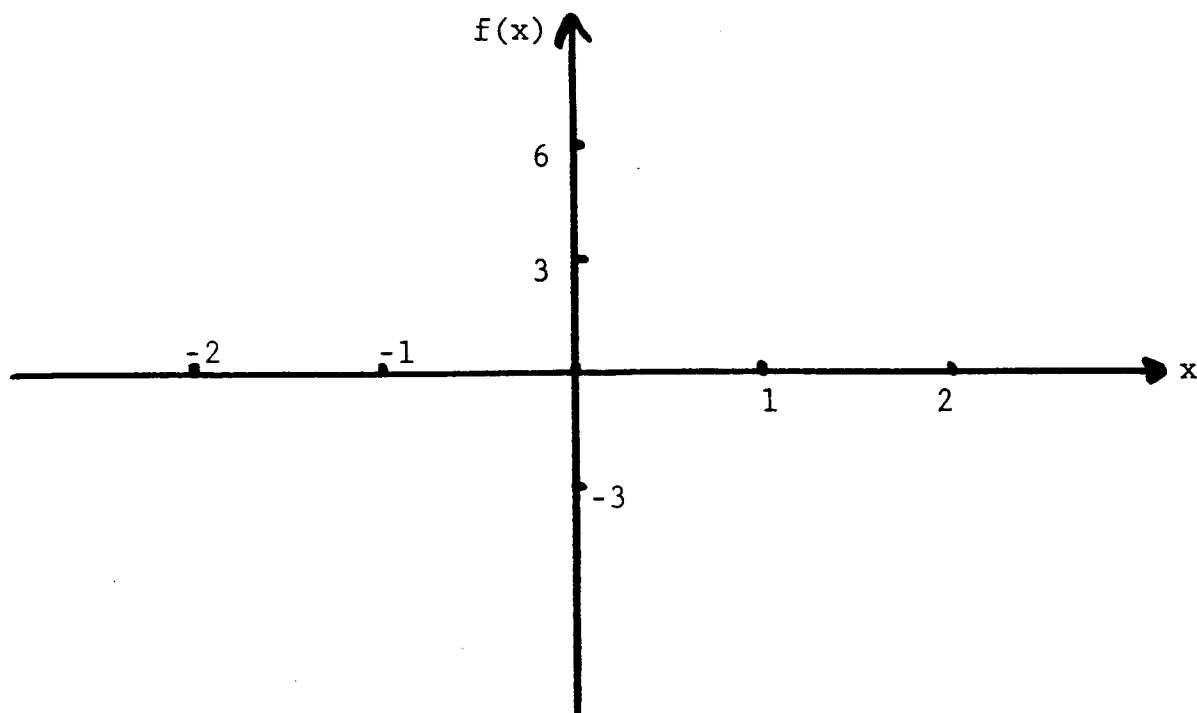
Question	Score
1	
2	
3	
4	
5	
Total	

Each question is worth 10 points.
No calculators are allowed.
Justify your answers carefully.
No partial credit can be given for unsubstantiated answers.

[1] Sketch the graph of the function

$$f(x) = x^2 + \frac{2}{x}.$$

Label all intercepts, extrema, and points of inflection (give their co-ordinates), and pay attention to limits and asymptotes.



[2] Find two positive numbers whose sum is 12 and such that the product of one of the numbers with the square of the other is as large as possible.

[3] An observer on a straight beach is watching a hovercraft which is travelling parallel to the shore, two miles out to sea, at a speed of thirty miles per hour. At what rate is the observer's head turning when the hovercraft passes directly in front of the observer?

[4] (a) Find the following limit (if it exists):

$$\lim_{x \rightarrow 0} \frac{x^2}{1 - \cos(3x)}.$$

(b) The graph of a certain function f has slope $\sin x + 7x^{12} + 4$ at every point (x, y) on the graph, and contains the point $(0, \pi)$. Find the function f .

[4] (c) Let f be a function such that $f'(a)$ and $f''(a)$ exist at a point a . Find

$$\lim_{h \rightarrow 0} \frac{f(a+h) - f(a-h)}{2h}.$$

[5] Consider a function $f(x)$ which is continuous on the closed interval $[p, q]$ and differentiable on the open interval (p, q) . We are told that $f(p) = f(q) = 0$, and that the graph of $f(x)$ is concave down on (p, q) . When answering the following questions you may use the Mean Value Theorem.

(a) Prove that the function $f(x)$ has a critical point r in the interval (p, q) .

(b) What is the sign of $f'(x)$ if x is less than r ? What if x is greater than r ?

[5] (c) What can you say about the sign of $f(x)$ for x in (p, q) ?