1. (10 points). True or false, no explanation necessary.

a) T F

If $(c, f(c))$ is a critical point of the function f, then f necessarily has a local extremum at c.

b) T F

$$\lim_{x \to 0} \frac{\sqrt{4+x}-2}{x} = \frac{1}{4} = \frac{(\sqrt{4+x}-2)(\sqrt{4+x}+2)}{x(\sqrt{4+x}+2)} = \frac{4+x-4}{x\sqrt{4+x}+2}$$

$$\log_2 x = \cancel{\ln_2} x^{1/\ln 2} = \frac{1}{\ln 2} \cancel{\ln} x$$

c) T F

$$e^{\log_2(x)} = x^{1/\ln(2)}$$

d) T F

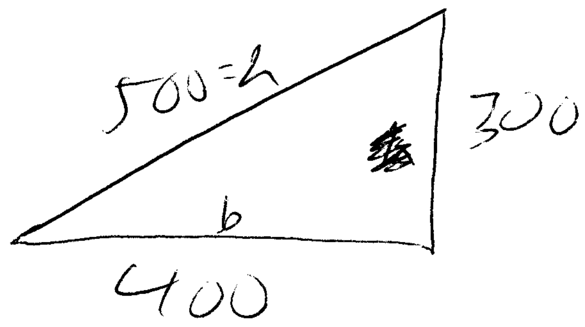
If $f'(-3) = 4$ and $f(x)$ is always concave up, then $\lim_{x\to\infty} f(x) = \infty$.

e) T F

$f(x) = \dfrac{1-9x^2}{3-4x+5x^2}$ has a horizontal asymptote at $y = 1/3$.

$$\frac{\frac{1}{x^2} - 9}{\frac{3}{x^2} - \frac{4}{x} + \cancel{5}} = \frac{-9}{5}$$

2. (10 points). A girl is flying a kite at a constant height of 300 ft. When the girl has let out 500 ft. of string, the wind is moving the kite horizontally at 25 ft./sec. How fast must she let out the kite string at this time?



$b^2 + 300^2 = h^2$

$2bb' + 0 = 2hh'$

$2 \cdot 400 \cdot 25 = 2 \cdot 500 \cdot h'$

$\frac{4}{5} \cdot 25 = h' = 20$

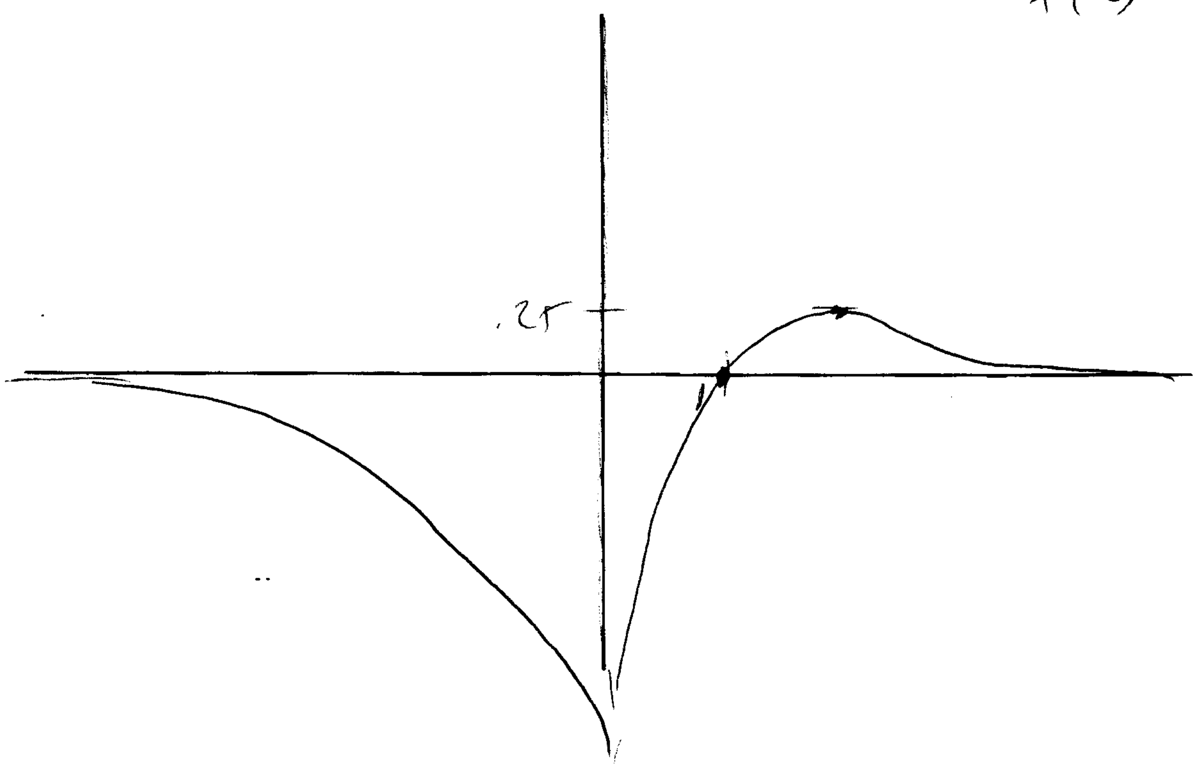
$$y' = \frac{1 \cdot x^2 - 2x(x-1)}{x^4} = \frac{x^2 - 2x^2 + 2x}{x^4}$$

$$= \frac{-x^2 + 2x}{x^4} = \frac{-x+2}{x^3} = 0 \text{ when } x = 2$$

3. (10 points).

a) Graph the function $y = (x-1)/x^2$ in the space provided. Make sure to label carefully all occurrences of relative extrema, inflection points, x and y-intercepts, and asymptotes.

$f(2) = .25$



$k < 0$, 2 solutions
$k = 0$, 1 solution
$k > 0$, 2 solutions

b) How many solutions does the equation $(x-1)/x^2 = k$ have for an arbitrary constant k? (You may have to distinguish several cases.)

1998 Midterm

4. $T = \text{Temperature} = T(t)$ $\quad T(0) = 30$

$T(t) = 30e^{-.03t}$

When does $T(t) = 5$?

$5 = 30e^{-.03t}$

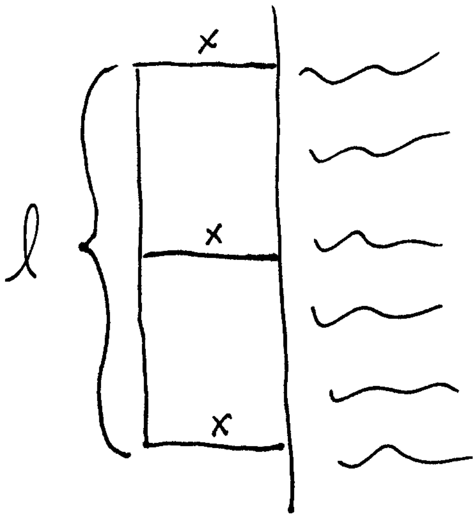
$\frac{1}{6} = e^{-.03t}$

$\ln\left(\frac{1}{6}\right) = -.03t$

$-1.79 = -.03t$

$\frac{179}{3} = t$ (in minutes)

5.



$3x + l = 300 \quad l = 300 - 3x$

$\text{Area} = A = xl = 300x - 3x^2$

$\frac{dA}{dx} = 300 - 6x = 0$

$x = 50$ is a max or min of the area

$x = 50$ then $l = 150$

$A = 7500$

Check if $x = 50$, $l = 150$ is max or min:

$A = 297$ if $x = 1$, $l = 297$ so

$x = 50$, $l = 150$, being the only critical point, must be ~~a~~ maximum