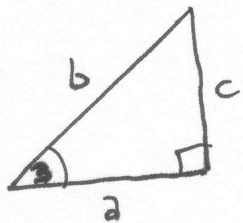


2000 Midterm

$$1. a) \frac{\log_5 16 \cdot \log_2 9}{\log_5 3} = \frac{\log_5 (2^4) \cdot \log_2 (3^2)}{\log_5 3}$$

$$= \frac{8 \log_5 2 \cdot \log_2 3}{\log_5 3} = 8 \log_3 2 \cdot \log_2 3 = 8$$

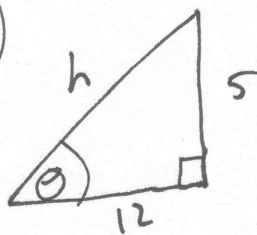
b)



$$\cos \theta = \frac{a}{b}$$

$$\sin^{-1}\left(\frac{a}{b}\right) = \frac{\pi}{2} - \theta = \arcsin(\cos(\theta))$$

c)



$$h = \sqrt{5^2 + 12^2} = 13$$

$$\theta = \arctan\left(\frac{5}{12}\right)$$

$$\cos \theta = \frac{12}{13}$$

$$\cos\left(\arctan\left(\frac{5}{12}\right)\right) = \frac{12}{13}$$

2.

$$a) \frac{d}{dx} (\ln \arccos(x)) = \frac{1}{\arccos(x)} \cdot (\arccos(x))' =$$

$$= \frac{1}{\arccos(x)} \cdot \left(-\frac{1}{\sqrt{1-x^2}} \right) = \boxed{-\frac{1}{\sqrt{1-x^2} \cdot \arccos(x)}}$$

$$b) \frac{d}{dx} (\arctan(x))^x$$

$$\text{let } (\arctan(x))^x = y \quad (\Rightarrow y' = \frac{d}{dx} (\arctan(x))^x)$$

$$\Rightarrow \ln [(\arctan(x))^x] = \ln y$$

$$\Rightarrow x \cdot \ln(\arctan(x)) = \ln y$$

$$\Rightarrow \ln(\arctan(x)) + x \cdot \frac{(\arctan(x))'}{\arctan(x)} = \frac{y'}{y}$$

$$\Rightarrow \boxed{y' = \left[\ln(\arctan(x)) + \frac{x}{(1+x^2) \cdot \arctan(x)} \right] \cdot (\arctan(x))^x}$$

$$c) y^3 + y \cdot \sin x - 1 = 0$$

$$\Rightarrow 3y^2 \cdot \frac{dy}{dx} + \frac{dy}{dx} \cdot \sin x + y \cdot \cos x = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{-y \cdot \cos x}{3y^2 + \sin x}$$

$$\text{at } x=0 \Rightarrow y^3 - 1 = 0 \Rightarrow y = 1$$

$$\Rightarrow \frac{dy}{dx} \Big|_{(0,1)} = \frac{-1 \cdot 1}{3 \cdot 1 + 0} = \boxed{-\frac{1}{3}}$$