Math 1a Midterm 2 Fall 2002 Solutions

(5) (a) $\lim_{x \to \pi/2} \frac{\sin x}{x} = \frac{\sin \pi/2}{\pi/2} = \frac{1}{\pi/2} = \boxed{2/\pi}$

(b) $\lim_{x \to 0} \left(\frac{1}{\sin 2x} - \frac{1}{2x} \right) = \text{"} \infty - \infty \text{"}$

$\overset{L}{\ominus}$ denotes use of L'Hôpital's Rule

$= \lim_{x \to 0} \frac{2x - \sin 2x}{2x \sin 2x} = \text{"} \frac{0}{0} \text{"}$

$\overset{L}{\ominus} \lim_{x \to 0} \frac{2 - 2\cos 2x}{(2x)(2\cos 2x) + (2)(\sin 2x)} = \text{"} \frac{0}{0} \text{"}$

$\overset{L}{\ominus} \lim_{x \to 0} \frac{+4\sin 2x}{(2x)(-4\sin 2x) + (2)(2\cos 2x) + 4\cos 2x} = \frac{0}{0+4+4}$

$= \frac{0}{8} = \boxed{0}$

(c) $\lim_{x \to 0^+} (e^x - 1)^{1/\ln x} = \text{"} 0^0 \text{"}$

$J = \lim_{x \to 0^+} (e^x - 1)^{1/\ln x}$

$\ln J = \ln \left(\lim_{x \to 0^+} (e^x - 1)^{1/\ln x} \right) = \lim_{x \to 0} \ln (e^x - 1)^{1/\ln x}$

$= \lim_{x \to 0} \frac{1}{\ln x} \ln(e^x - 1) = \lim_{x \to 0} \frac{\ln(e^x - 1)}{\ln x} = \text{"} \frac{\infty}{\infty} \text{"}$

$\overset{L}{\ominus} \lim_{x \to 0} \frac{\frac{e^x}{e^x - 1}}{\frac{1}{x}} = \lim_{x \to 0^+} \frac{x e^x}{e^x - 1} = \text{"} \frac{0}{0} \text{"}$

$$\overset{L}{=} \lim_{x\to 0^+} \frac{xe^x + e^x}{e^x} = \lim_{x\to 0^+} \frac{x+1}{1} = 1$$

$$\Rightarrow \ln J = 1$$

$$J = e^1 = \boxed{e}$$

(d) $\lim_{x\to\infty} \left(1+\frac{1}{3x}\right)^x = "1^\infty"$

$$J = \lim_{x\to\infty} \left(1+\frac{1}{3x}\right)^x$$

$$\ln J = \ln\left(\lim_{x\to\infty} \left(1+\frac{1}{3x}\right)^x\right) = \lim_{x\to\infty} \ln\left(1+\frac{1}{3x}\right)^x$$

$$= \lim_{x\to\infty} x \ln\left(1+\frac{1}{3x}\right) = "\infty \cdot 0"$$

$$= \lim_{x\to\infty} \frac{\ln\left(1+\frac{1}{3x}\right)}{\frac{1}{x}} = "\frac{0}{0}"$$

$$\overset{L}{=} \lim_{x\to\infty} \frac{\frac{-\frac{1}{3}x^{-2}}{1+\frac{1}{3x}}}{-x^{-2}} = \lim_{x\to\infty} \frac{-\frac{1}{3}x^{-2}}{\left(1+\frac{1}{3x}\right)(-x^{-2})} = \lim_{x\to\infty} \frac{1/3}{1+1/3x} = 1/3$$

$$\Rightarrow \ln J = 1/3 \Rightarrow J = \boxed{e^{1/3}}$$

(e) $\lim_{x\to 0} \frac{\sin^2 x}{\tan^2 x} = \lim_{x\to 0} \frac{\sin^2 x}{\frac{\sin^2 x}{\cos^2 x}} = \lim_{x\to 0} \sin^2 x \cdot \frac{\cos^2 x}{\sin^2 x}$

$$= \lim_{x\to 0} \cos^2 x = \boxed{1}$$