Math 1a Fall 2004
Final Exam Review
Solutions

1. 67 people

2. $x_3 = \dfrac{2387}{2000}$

3. Note that $a(t) = -32$ gives the acceleration (in feet per second per second) of the package $t = s$ after it is released. Integrating this gives

$$v(t) = -32t + C$$

for the velocity of the package. Since the package initially has the same upward velocity as the balloon, we have that

$$12 = v(0) = -32(0) + C = 12$$

and so $C = 12$. Thus,

$$v(t) = -32t + 12.$$

Integrating gives

$$h(t) = -16t^2 + 12t + D$$

for the height of the package. Since the package has height 80 when it is released, we have that

$$80 = h(0) = -16(0)^2 + 12(0) + D = D$$

and so $D = 80$. Thus

$$h(t) = -16t^2 + 12t + 80.$$

To determine when the package reaches the ground, we determine when its height is 0. Thus

$$0 = h(t) = -16t^2 + 12t + 80$$

or

$$0 = 4t^2 - 3t - 20.$$

Using the quadratic formula, we find that

$$t = \frac{3 \pm \sqrt{329}}{8} \approx 2.64229 \text{ and } -1.89229.$$

Thus the package reaches the ground after approximately 2.64 seconds.

4. (a) 2π

   (b) 2π

   (c) $\displaystyle\lim_{n\to\infty} \sum_{i=1}^{n} \left(\sin\left(-\pi + \frac{2\pi i}{n} \right) + 1 \right) \frac{2\pi}{n}$

   (d) 2π

5. $\dfrac{37}{12}$

6. $\dfrac{1}{3}$

7. Evaluate each of the following integrals.

   (a) 4
   (b) 0
   (c) $\dfrac{1}{6}$
   (d) $\dfrac{4}{\sqrt{\cos\left(\sqrt{\theta}\right)}} + C$

8. (a) $\displaystyle\int_{10}^{11} r(t)\,dt - \int_{10}^{11} 30\,dt \approx 7$
   (b) $\displaystyle\int_{10}^{14} r(t)\,dt - \int_{10}^{14} 30\,dt \approx 45$
   (c) $\displaystyle\int_{8}^{16} r(t)\,dt \approx 225$
   (d) Note that at 4 PM, about 45 people are in line. Between 4 PM and 6 PM, an additional $\displaystyle\int_{14}^{16} r(t)\,dt \approx$ 30 people arrive for flu shots. Since the clinic can only serve 30 people per hour, a total of about $(45+30) - 2*30 = 15$ people are still in line at 6 PM. Since about 225 people arrived for flu shots and about 15 of them are not served, the clinic served about 210 people during the day.

   How can we represent this in terms of definite integrals? Since the line never goes away after 10 AM, the clinic serves 30 people per hour between 10 AM and 4 PM. Since the clinic also serves all people who arrive between 8 AM and 10 AM, we have that the number of people actually served by the clinic is

$$\int_{8}^{10} r(t)\,dt + \int_{10}^{16} 30\,dt.$$