Mathematics 1a, Tangent Problem Worksheet Solutions

Alexander Ellis

October 1, 2004

1. The definition of a circle's tangent is that it touches the circle exactly once. But for an arbitrary curve, such as the one in figure 1b on page 95, we run into a problem: The line at point P which seems to best represent the slope hits the curve again in another place. There are lines, such as ℓ, which only hit the curve once, but for an arbitrary curve there is no guarantee the line will in any way represent the direction of the curve at the point.

To summarize, the definition of a tangent line at P as the line which intersects the curve only at P becomes problematic for at least three reasons: there may be more than one possible choice for this line, none of these lines necessarily represent the direction of the curve, and the one which does represent the direction isn't necessarily one of these lines that intersects only once.

2. The slope of a line is the ratio

$$\frac{\Delta y}{\Delta x}$$

between *two* points. If the only known point is P, we cannot compute a slope.

3. The secant line PQ is the line which passes through points P and Q, both of which lay on the curve. Thus the secant line intersects the curve at (at least) two points.

4. The equation

$$m_{PQ} = \frac{x^2 - 1}{x - 1}$$

is the limit of the slope of secant lines PQ as the point Q approaches the point P.

$$m_{PQ} = \frac{(x_Q)^2 - (x_P)^2}{x_Q - x_P} = \frac{x^2 - 1}{x - 1}$$

where the point P is $(1, 1)$.

5. The table was:

x	$m_{PQ} = \dfrac{x^2 - 1}{x - 1}$
2	3
1.5	2.5
1.1	2.1
1.01	2.01
1.001	2.001
0	1
0.5	1.5
0.9	1.0
0.99	1.99
0.999	1.999

6. The four added rows are:

x	m_{PQ}
1.0001	2.0001
1.00001	2.00001
0.9999	1.9999
0.99999	1.99999

7. The limit represents the limit of the secant slopes as the point Q approaches the point P. Numerically, these slopes were found to approach 2 as x approached 1. Given this numeric evidence, it is likely the limit is equal to 2.