Math 1a Fall 2004
Worksheet: The Derivative at a Number

1. Which of the following limits are equal to $g'(3)$? (Sketching and labeling the graph for a function g might be helpful as you answer this question.)

   (a) $\lim_{h \to 0} \frac{g(3+h) - g(3)}{h}$

   (b) $\lim_{x \to 0} \frac{g(x) - g(3)}{x - 3}$

   (c) $\lim_{x \to 3} \frac{g(x) - g(3)}{x - 3}$

   (d) $\lim_{\Delta x \to 3} \frac{g(3 + \Delta x) - g(3)}{\Delta x}$

   (e) $\lim_{\Delta x \to 0} \frac{g(3 + \Delta x) - g(3)}{\Delta x}$

   (f) $\lim_{\Delta x \to 0} \frac{g(3) - f(3 + \Delta x)}{\Delta x}$

2. If an arrow is shot upward on the moon with a velocity of 58 meters per second, its height in meters after t seconds is given by $h(t) = 58t - 0.83t^2$.

   (a) Find the velocity of the arrow after one second.

   (b) Find the velocity of the arrow when $t = a$.

   (c) When will the arrow hit the moon?

   (d) With what velocity will the arrow hit the moon?

3. Between 1940 and 1995 the size of the average farm in America increased from 174 acres to 469 acres. Suppose that $A(t)$ gives the average number of acres of an American farm t years after 1940. Note that $A(t)$ is an increasing function. Use the fact that average farm size increased much more dramatically in the 1950s than in the 1980s to answer the following questions.

   (a) Which is larger, $A(12)$ or $A(43)$?

   (b) Which is larger, $A'(12)$ or $A'(43)$?

4. Sketch the graph of a single function that satisfies all of the following properties.

   (a) $\lim_{x \to -3^-} f(x) = \infty$

   (b) $\lim_{x \to -3^+} f(x) = -\infty$

   (c) $\lim_{x \to \pm\infty} f(x) = 0$

   (d) $f(0) = 0$

   (e) $f'(0) = 1$

   (f) $f'(2)$ does not exist

   (g) $f(x) < 0$ for all $x > 3$

   (h) $f'(4) = 0$