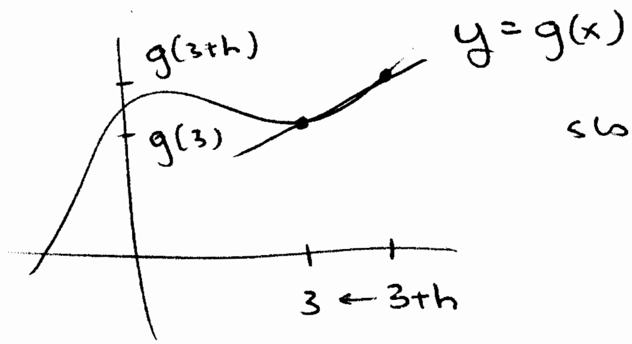
The Derivative at a Number Solutions

① (a) (e) (f)

For example,



slope of secant line

$$= \frac{g(3+h) - g(3)}{3+h-3}$$

$$= \frac{g(3+h) - g(3)}{h}$$

Thus, the slope of tangent line $= g'(3) = \lim_{h \to 0} \frac{g(3+h) - g(3)}{h}$

② (a) $f(t) = 58t - 0.83t^2$

$$f'(1) = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h} = \lim_{h \to 0} \frac{58(1+h) - .83(1+h)^2 - 57.17}{h}$$

$$= \lim_{h \to 0} \frac{58 + 58h - .83 - 1.66h - .83h^2 - 57.17}{h}$$

$$= \lim_{h \to 0} \frac{58h - 1.66h - .83h^2}{h} = \lim_{h \to 0} 58 - 1.66 - .83h$$

$= 56.34$ m/s

(b) $f'(a) = \lim_{h \to 0} \dfrac{f(a+h) - f(a)}{h}$

$= \lim_{h \to 0} \dfrac{58(a+h) - .83(a+h)^2 - (58a - .83a^2)}{h}$

$= \lim_{h \to 0} \dfrac{\cancel{58a} + 58h - \cancel{.83a^2} - 1.66ah - .83h^2 - \cancel{58a} + \cancel{.83a^2}}{h}$

$= \lim_{h \to 0} \dfrac{h(58 - 1.66a - .83h)}{h} = \lim_{h \to 0} 58 - 1.66a - .83h$

$= \boxed{58 - 1.66a}$

(c) $0 = f(t) = 58t - .83t^2 = t(58 - .83t)$

$t = 0$ or $\dfrac{58}{.83} \approx 69.88$ s

(d) $f'(69.88) = 58 - 1.66(69.88) \approx \boxed{58 \text{ m/s}}$

3 (a) $A(43) > A(12)$ since farms were bigger in 1983 than they were in 1952 since $A(t)$ is an increasing ftn.

(b) $A'(12) > A'(43)$ since farm size changed more rapidly in 1952 than it did in 1983.

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