

**Extreme Value Theorem  
Mean Value Theorem  
Solutions**

1. *Answer: (a).* This problem emphasizes one important application of the EVT which is at the same time a very geometric result; that we can put a continuous function on a closed interval inside a box! On a closed interval,  $m \leq f(x) \leq M$ , so if we take  $A = \max\{|m|, |M|\}$ , then on this closed interval,  $f$  fits in this sort of box. This idea will ultimately show up in finding bounds for integrals.
2. *Answer: (d).* First answer this question in the easiest form, discussing instantaneous and average velocity - then go back and try to answer again in a way that allows the application of the mean value theorem easily. The fact that (c) is also true, can be shown by using a graph. This is probably the hardest part of the problem.
3. *Answer: False.* This emphasizes the differentiability hypotheses when using the MVT (Note that  $f(x)$  is not differentiable at 0).
4. *Answer: (c).*  $\frac{A_2 - A_1}{r_2 - r_1} = \frac{\pi(r_2^2 - r_1^2)}{r_2 - r_1} = \pi(r_2 + r_1)$ . On the other hand, by MVT,  $\frac{A_2 - A_1}{r_2 - r_1} = 2\pi r$  for some  $r \in (r_1, r_2)$ . Thus,  $A_2 - A_1 = 2\pi r(r_2 - r_1)$ . This problem is a fairly simple application of the MVT, but the fact that you can get the answer to be exactly the area of that rectangle might surprise you. This problem will be a good reference later on, when we are trying to prove the evaluation part of the Fundamental Theorem of Calculus.