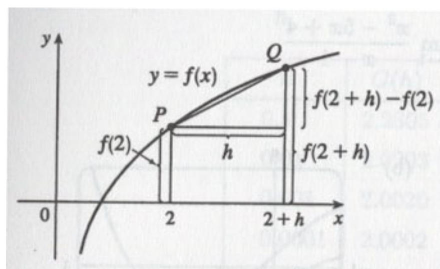
Mathematics 1a, Section 2.7 Solutions

Alexander Ellis

October 14, 2004

1.



The line from $P(2, f(2))$ to $Q(2+h, f(2+h))$ is the line that has slope $\frac{f(2+h)-f(2)}{h}$.

14.

$$\begin{aligned}
f'(a) &= \lim_{h\to 0}\frac{f(a+h)-f(a)}{h} \\
&= \lim_{h\to 0}\frac{[(a+h)^4-5(a+h)]-(a^4=5a)}{h} \\
&= \lim_{h\to 0}\frac{(a^4+4a^3h+6a^2h^2+4ah^3+h^4-5a-5h)-(a^4-5a)}{h} \\
&= \lim_{h\to 0}\frac{4a^3h+6a^2h^2+4ah^3+h^4-5h}{h} \\
&= \lim_{h\to 0}\frac{h(4a^3+6a^2h+4ah^2+h^3-5)}{h} \\
&= \lim_{h\to 0}(4a^3+6a^2h+4ah^2+h^3-5) = 4a^3-5
\end{aligned}$$

22. By Equation 3,

$$\lim_{x\to\pi/4}\frac{\tan x - 1}{x-\pi/4} = f'(\pi/4)$$

where $f(x) = \tan x$.

34. For 1910, we will average the difference quotients obtained using the years 1900 and 1920. Let

$$A = \frac{E(1900) - E(1910)}{1900 - 1910} = \frac{48.3 - 51.1}{-10} = 0.28$$
$$B = \frac{E(1920) - E(1910)}{1920 - 1910} = \frac{55.2 - 51.1}{10} = 0.41$$

Then

$$E'(1910) = \lim_{t \to 1910} \frac{E(t) - E(1910)}{t - 1910} \approx \frac{A + B}{2} = 0.345$$

Thus the life expectancy at birth was increasing at about 0.345 year/year in 1910.
For 1950, we use data from 1940 and 1960 in a similar fashion, and obtain

$$E'(1950) \approx (0.31 + 0.10)/2 = 0.205$$

so the life expectancy at birth was increasing at about 0.205 year/year in 1950.

36. Since $f(x) = x^2 \sin(1/x)$ when $x \neq 0$ and $f(0) = 0$, we have

$$f'(0) = \lim_{h \to 0} \frac{f(0+h) - f(0)}{h} = \lim_{h \to 0} \frac{h^2 \sin(1/h) - 0}{h} = \lim_{h \to 0} h \sin(1/h)$$

Since $-1 \leq \sin \frac{1}{h} \leq 1$, we have

$$-|h| \leq |h| \sin \frac{1}{h} \leq |h| \Rightarrow -|h| \leq h \sin \frac{1}{h} \leq |h|$$

Because $\lim_{h \to 0}(-|h|) = 0$ and $\lim_{h \to 0} |h| = 0$, we know that

$$\lim_{h \to 0} h \sin \frac{1}{h} = 0$$

by the Squeeze Theorem. Thus, $f'(0) = 0$.