Mathematics 1a, Section 2.8 Solutions

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1. It appears that f is an odd function, so f' will be an even function - that is, $f'(-a) = f'(a)$.

a. $f'(-3) \approx 1.5$

b. $f'(-2) \approx 1$

c. $f'(-1) \approx 0$

d. $f'(0) \approx -4$

e. $f'(1) \approx 0$

f. $f'(2) \approx 1$

g. $f'(3) \approx 1.5$

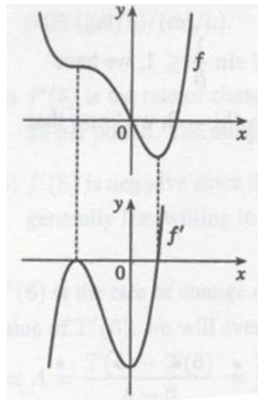
3. a. $a' =$ II, since from left to right, the slopes of the tangents to a start out negative, become 0, then positive, then 0, then negative again. The actual function values in graph II follow the same pattern.

b. $b' =$ IV, since from left to right, the slopes of the tangents to graph b start out at a fixed positive quantity, then suddenly become negative, then positive again. The discontinuities in graph IV indicate sudden changes in the slopes of the tangents.

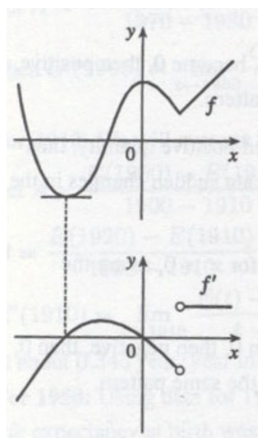
c. $c' =$ I, since the slopes of the tangents to graph c are negative for $x < 0$ and positive for $x > 0$, as are the function values of graph I.

d. $d' =$ III, since from left to right, the slopes of the tangents to graph d are positive, then 0, then negative, then 0, then positive, then 0, then negative again, and the function values in graph III follow the same pattern.

4. For this and **10**, first plot x-intercepts on the graph of f' for any horizontal tangents on the graph of f. Look for any corners on the graph of f - there will be a continuity on the graph of f'. On any interval where f has a tangent with positive (or negative) slope, the graph of f' will be positive (or negative). If the graph of the function is linear, the graph of f' will be a horizontal line.



10. Follow the suggestions in **4**.



24.

$$
\begin{aligned}
g'(x) &= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \\
&= \lim_{h\to 0} \frac{\frac{3+(x+h)}{1-3(x+h)} - \frac{3+x}{1-3x}}{h} \\
&= \lim_{h\to 0} \frac{(3+x+h)(1-3x)-(3+x)(1-3x-3h)}{h(1-3x-3h)(1-3x)} \\
&= \lim_{h\to 0} \frac{(3-9x+x-3x^2+h-3hx)-(3-9x-9h+x-3x^2-3hx)}{h(1-3x-3h)(1-3x)} \\
&= \lim_{h\to 0} \frac{10h}{h(1-3x-3h)(1-3x)} \\
&= \lim_{h\to 0} \frac{10}{(1-3x-3h)(1-3x)} = \frac{10}{(1-3x)^2}
\end{aligned}
$$

28. a. $U'(t)$ is the rate at which the unemployment rate is changing with respect to time. Its units are percent per year.

b. To find $U'(t)$, we use

$$\lim_{h\to 0}\frac{U(t+h)-U(t)}{h}\approx\frac{U(t+h)-U(t)}{h}$$

for small values of h. For 1989:

$$U'(1989)=\frac{U(1990)-U(1989)}{1990-1989}=\frac{5.6-5.3}{1}=0.30$$

For 1990, we estimate $U'(1990)$ by using $h=-1$ and $h=1$, and then averaging the two results to obtain a final estimate.

$$h=-1\Rightarrow U'(1990)\approx\frac{U(1989)-U(1990)}{1989-1990}=\frac{5.3-5.6}{-1}=0.30$$
$$h=1\Rightarrow U'(1990)\approx\frac{U(1991)-U(1990)}{1991-1990}=\frac{6.8-5.6}{1}=1.20$$

so we estimate that $U'(1990)\approx\frac{1}{2}(0.30+1.20)=0.75$.

t	1989	1990	1991	1992	1993	1994	1995	1996	1997	1998
$U'(t)$	0.30	0.75	0.95	0.05	-0.70	-0.65	-0.35	-0.35	-0.45	-0.40

30. a. $S'(t)$ is the rate at which the smoking rate is changing with respect to time. Its units are percent per year.

b. To find $S'(t)$, we use

$$\lim_{h\to 0}\frac{S(t+h)-S(t)}{h}\approx\frac{S(t+h)-S(t)}{h}$$

for small values of h. For 1980:

$$S'(1980)\approx\frac{S(1982)-S(1980)}{1982-1980}=\frac{21.0-21.4}{2}=\frac{-0.4}{2}=-0.20$$

For 1982, we estimate $S'(1982)$ by using $h=-2$ and $h=2$, and then averaging the two results to obtain a final estimate.

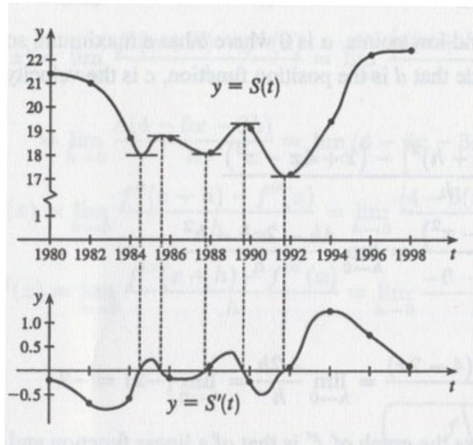
$$h=-2\Rightarrow S'(1982)\approx\frac{S(1980)-S(1982)}{1980-1982}=\frac{21.0-21.4}{2}=\frac{-0.4}{2}=-0.20$$
$$h=2\Rightarrow S'(1982)=\frac{S(1984)-S(1982)}{1984-1982}=\frac{18.7-21.0}{2}=-1.15$$

So we estimate that

$$S'(1982)\approx\frac{1}{2}(-0.20-1.15)=-0.675$$

t	1980	1982	1984	1986	1988	1990	1992	1994	1996	1998
$S'(t)$	-0.20	-0.675	-0.575	-0.15	0.10	-0.225	0.075	1.25	0.75	0.10

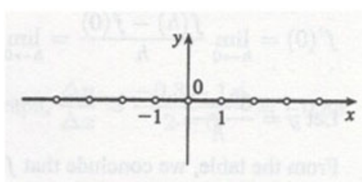
c.



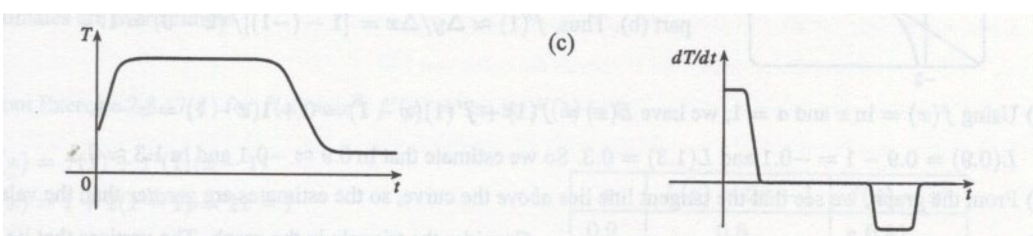
d. We could get more accurate values for $S'(t)$ by obtaining data for the odd-numbered years.

38. a must be the jerk since none of the graphs are 0 at its high and low points. a is 0 where b has a maximum, so $b' = a$. b is 0 where c has a maximum, so $c' = b$. We conclude that d is the position function, c is the velocity, b is the acceleration, and d is the jerk.

46. $f(x) = [x]$ is not continuous at any integer n, so f is not differentiable at n by the contrapositive of Theorem 4. If a is not an integer, then f is constant on an open interval containing a, so $f'(a) = 0$. Thus, $f'(x) = 0$, for x not an integer.



48.



b. The initial temperature of the water is close to room temperature because of the water that was in the pipies. When the water from the hot water tank starts coming out, $\frac{dT}{dt}$ is large and positive as T increases to the temperature of the water in the tank. In the next phase, $\frac{dT}{dt} = 0$ as the water comes out at a constant, high temperature. After some time, $\frac{dT}{dt}$ becomes small and negative as the contents of the hot water tank are exhausted. Finally, when the hot water has run out, $\frac{dT}{dt}$ is once again 0 as the water maintains its cold temperature.