Mathematics 1a, Section 4.2 Solutions

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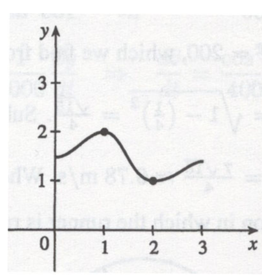
2. a. The Extreme Value Theorem.

b. See the Closed Interval Method.

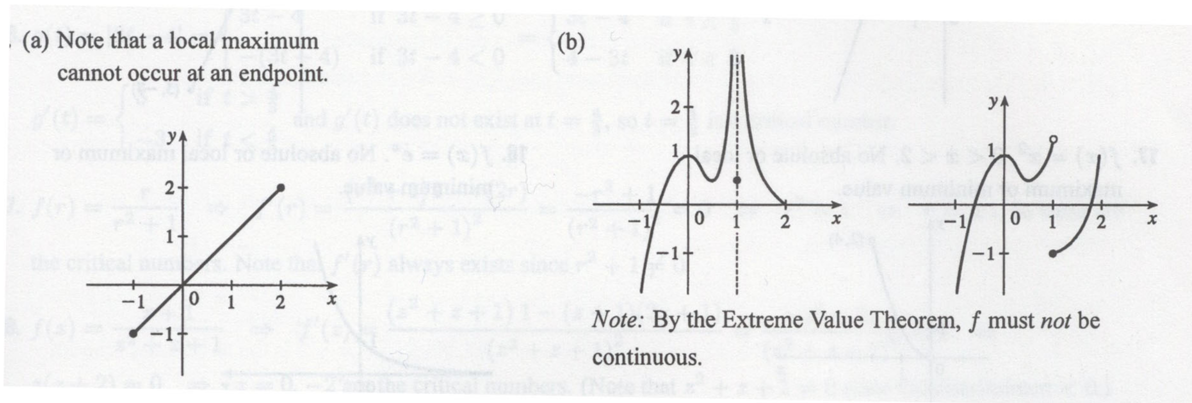
4. Absolute maximum at e; absolute minimum at t; local maxima at c, e, and s; local minima at b, c, d, and r; neither a maximum nor a minimum at a.

6. Absolute maximum value if $f(7) = 5$; absolute minimum value is $f(1) = 0$; local maximum values are $f(0) = 2$, $f(3) = 4$, and $f(5) = 3$; local minimum values are $f(1) = 0$, $f(4) = 2$, and $f(6) = 1$.

8.



12. a.



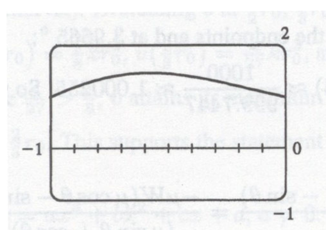
40. $f(x) = \frac{x}{x^2+4}$, on the interval $[0, 3]$.

$$f'(x) = \frac{(x^2+4)1 - x(2x)}{(x^2+4)^2} = \frac{4-x^2}{(x^2+4)^2} = 0$$

$$x = \pm 2$$

However, -2 is not in the interval $[0, 3]$. $f(0) = 0$, $f(2) = 1/4 = 0.25$, $f(3) = 3/13 \approx 0.23$. So $f(2) = 1/4$ is the absolute maximum and $f(0) = 0$ is the absolute minimum.

48. a.



From the graph, it appears that the absolute maximum value is about $f(-0.58) = 1.47$, and the absolute minimum value is about $f(-1) = f(0) = 1.00$; that is, at both endpoints.

b.

$$f(x) = e^{x^3-x}$$

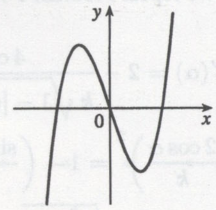
$$f'(x) = e^{x^3-x}(3x^2-1)$$

So $f'(x) = 0$ on $[-1, 0]$ implies that $x = -\sqrt{1/3}$. $f(-1) = f(0) = 1$ (minima) and $f(-\sqrt{1/3}) = e^{-\sqrt{3}/9+\sqrt{3}/3} = e^{2\sqrt{3}/9}$ (maximum).

56. a. $f(x) = ax^3 + bx^2 + cx + d$, and $a \neq 0$. So $f'(x) = 3ax^2 + 2bx + c$ is a quadratic and hence has either 2, 1, or 0 real roots, so $f(x)$ has either 2, 1, or 0 critical numbers.

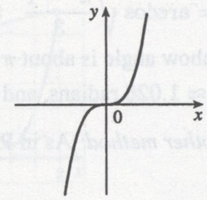
Case (i) (2 critical numbers): $f(x) = x^3 - 3x \Rightarrow f'(x) = 3x^2 - 3$,

so $x = -1, 1$ are critical numbers.



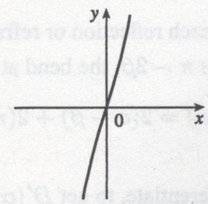
Case (ii) (1 critical number): $f(x) = x^3 \Rightarrow f'(x) = 3x^2$,

so $x = 0$ is the only critical number.



Case (iii) (no critical number): $f(x) = x^3 + 3x \Rightarrow f'(x) = 3x^2 + 3$,

so there are no real roots.



b. Since there are at most two critical numbers, it can have at most two local extreme values, and by (i) this can occur. By (iii) it can have no local extreme value. However, if there is only one critical number, then there is no local extreme value.