Name: ___________________ Class:___ Date: ______________

1. (a) As x approaches 2 (from the right or the left), the values of $f(x)$ become large.

   (b) As x approaches 1 from the right, the values of $f(x)$ become large negative.

   (c) As x becomes large, the values of $f(x)$ approach 5.

   (d) As x becomes large negative, the values of $f(x)$ approach 3.

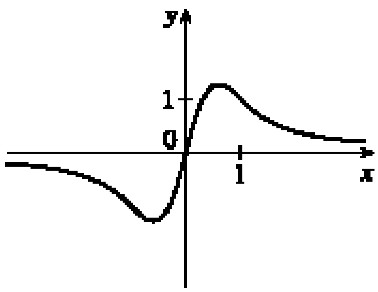
4. (a) $\lim_{x\to\infty} g(x) = 2$ (b) $\lim_{x\to-\infty} g(x) = -2$ (c) $\lim_{x\to3} g(x) = \infty$

   (d) $\lim_{x\to0} g(x) = -\infty$ (e) $\lim_{x\to-2^+} g(x) = -\infty$ (f) Vertical: $x = -2$, $x = 0$, $x = 3$; Horizontal: $y = -2$, $y = 2$

5. $f(0) = 0$, $f(1) = 1$,

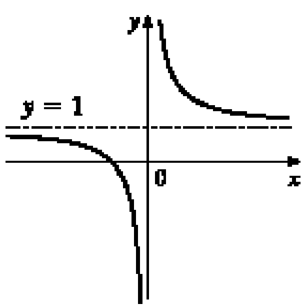
   $\lim_{x\to\infty} f(x) = 0$,

   f is odd



6. $\lim_{x\to0^+} f(x) = \infty$, $\lim_{x\to0^-} f(x) = -\infty$,

   $\lim_{x\to\infty} f(x) = 1$, $\lim_{x\to-\infty} f(x) = 1$



23. First, multiply the factors in the denominator. Then divide both the numerator and denominator by u^4.

$$\lim_{u\to\infty}\frac{4u^4+5}{(u^2-2)(2u^2-1)}=\lim_{u\to\infty}\frac{4u^4+5}{2u^4-5u^2+2}=\lim_{u\to\infty}\frac{\dfrac{4u^4+5}{u^4}}{\dfrac{2u^4-5u^2+2}{u^4}}=\lim_{u\to\infty}\frac{4+\dfrac{5}{u^4}}{2-\dfrac{5}{u^2}+\dfrac{2}{u^4}}$$

$$=\frac{\lim\limits_{u\to\infty}\left(4+\dfrac{5}{u^4}\right)}{\lim\limits_{u\to\infty}\left(2-\dfrac{5}{u^2}+\dfrac{2}{u^4}\right)}=\frac{\lim\limits_{u\to\infty}4+5\lim\limits_{u\to\infty}\dfrac{1}{u^4}}{\lim\limits_{u\to\infty}2-5\lim\limits_{u\to\infty}\dfrac{1}{u^2}+2\lim\limits_{u\to\infty}\dfrac{1}{u^4}}=\frac{4+5(0)}{2-5(0)+2(0)}=\frac{4}{2}=2$$

25. $$\lim_{x\to\infty}\left(\sqrt{9x^2+x}-3x\right)=\lim_{x\to\infty}\frac{\left(\sqrt{9x^2+x}-3x\right)\left(\sqrt{9x^2+x}+3x\right)}{\sqrt{9x^2+x}+3x}=\lim_{x\to\infty}\frac{\left(\sqrt{9x^2+x}\right)^2-(3x)^2}{\sqrt{9x^2+x}+3x}$$

$$=\lim_{x\to\infty}\frac{(9x^2+x)-9x^2}{\sqrt{9x^2+x}+3x}=\lim_{x\to\infty}\frac{x}{\sqrt{9x^2+x}+3x}\cdot\frac{1/x}{1/x}$$

$$=\lim_{x\to\infty}\frac{x/x}{\sqrt{9x^2/x^2+x/x^2}+3x/x}=\lim_{x\to\infty}\frac{1}{\sqrt{9+1/x}+3}=\frac{1}{\sqrt{9}+3}=\frac{1}{3+3}=\frac{1}{6}$$

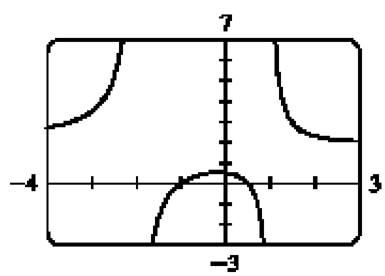
27. $\lim\limits_{x\to\infty}\cos x$ does not exist because as x increases $\cos x$ does not approach any one value, but oscillates between 1 and -1.

30. $\lim\limits_{x\to\infty}\tan^{-1}(x^2-x^4)=\lim\limits_{x\to\infty}\tan^{-1}(x^2(1-x^2))$. If we let $t=x^2(1-x^2)$, we know that $t\to-\infty$ as $x\to\infty$, since $x^2\to\infty$ and $1-x^2\to-\infty$. So $\lim\limits_{x\to\infty}\tan^{-1}(x^2(1-x^2))=\lim\limits_{t\to-\infty}\tan^{-1}t=-\frac{\pi}{2}$.

35. $$\lim_{x\to\infty}\frac{2x^2+x-1}{x^2+x-2}=\lim_{x\to\infty}\frac{\dfrac{2x^2+x-1}{x^2}}{\dfrac{x^2+x-2}{x^2}}=\lim_{x\to\infty}\frac{2+\dfrac{1}{x}-\dfrac{1}{x^2}}{1+\dfrac{1}{x}-\dfrac{2}{x^2}}=\frac{\lim\limits_{x\to\infty}\left(2+\dfrac{1}{x}-\dfrac{1}{x^2}\right)}{\lim\limits_{x\to\infty}\left(1+\dfrac{1}{x}-\dfrac{2}{x^2}\right)}$$

$$=\frac{\lim\limits_{x\to\infty}2+\lim\limits_{x\to\infty}\dfrac{1}{x}-\lim\limits_{x\to\infty}\dfrac{1}{x^2}}{\lim\limits_{x\to\infty}1+\lim\limits_{x\to\infty}\dfrac{1}{x}-2\lim\limits_{x\to\infty}\dfrac{1}{x^2}}=\frac{2+0-0}{1+0-2(0)}=2,$$ so $y=2$ is a horizontal asymptote.

$y=f(x)=\dfrac{2x^2+x-1}{x^2+x-2}=\dfrac{(2x-1)(x+1)}{(x+2)(x-1)}$, so $\lim\limits_{x\to-2^-}f(x)=\infty$, $\lim\limits_{x\to-2^+}f(x)=-\infty$, $\lim\limits_{x\to1^-}f(x)=-\infty$, and $\lim\limits_{x\to1^+}f(x)=\infty$. Thus, $x=-2$ and $x=1$ are vertical asymptotes. The graph confirms our work.



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40. Since the function has vertical asymptotes $x = 1$ and $x = 3$, the denominator of the rational function we are looking for must have factors $(x - 1)$ and $(x - 3)$. Because the horizontal asymptote is $y = 1$, the degree of the numerator must equal the degree of the denominator, and the ratio of the leading coefficients must be 1. One possibility is $f(x) = \dfrac{x^2}{(x-1)(x-3)}$.

45. $\lim\limits_{x\to\infty} \dfrac{5\sqrt{x}}{\sqrt{x-1}} \cdot \dfrac{1/\sqrt{x}}{1/\sqrt{x}} = \lim\limits_{x\to\infty} \dfrac{5}{\sqrt{1-(1/x)}} = \dfrac{5}{\sqrt{1-0}} = 5$ and

$\lim\limits_{x\to\infty} \dfrac{10e^x - 21}{2e^x} \cdot \dfrac{1/e^x}{1/e^x} = \lim\limits_{x\to\infty} \dfrac{10-(21/e^x)}{2} = \dfrac{10-0}{2} = 5$. Since $\dfrac{5\sqrt{x}}{\sqrt{x-1}} < f(x) < \dfrac{10e^x - 21}{2e^x}$, we have $\lim\limits_{x\to\infty} f(x) = 5$ by the Squeeze Theorem.

47. (a) After t minutes, $25t$ liters of brine with 30 g of salt per liter has been pumped into the tank, so it contains $(5000 + 25t)$ liters of water and $25t \cdot 30 = 750t$ grams of salt. Therefore, the salt concentration at time t will be $C(t) = \dfrac{750t}{5000+25t} = \dfrac{30t}{200+t}\ \dfrac{\text{g}}{\text{L}}$.

(b) $\lim\limits_{t\to\infty} C(t) = \lim\limits_{t\to\infty} \dfrac{30t}{200+t} = \lim\limits_{t\to\infty} \dfrac{30t/t}{200/t + t/t} = \dfrac{30}{0+1} = 30$. So the salt concentration approaches that of the brine being pumped into the tank.