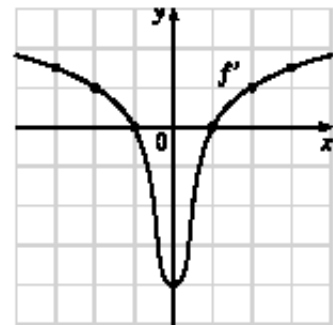


1. It appears that f is an odd function, so f' will be an even function—that is, $f'(-a) = f'(a)$.

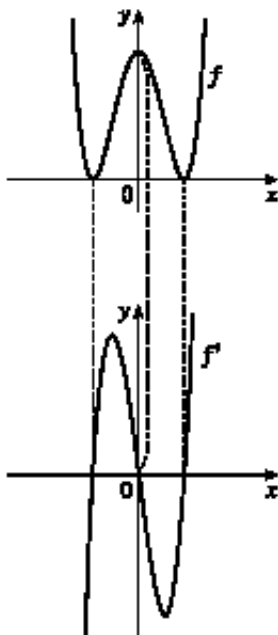
- (a) $f'(-3) \approx 1.5$ (b) $f'(-2) \approx 1$
 (c) $f'(-1) \approx 0$ (d) $f'(0) \approx -4$
 (e) $f'(1) \approx 0$ (f) $f'(2) \approx 1$
 (g) $f'(3) \approx 1.5$



3. (a)' = II, since from left to right, the slopes of the tangents to graph (a) start out negative, become 0, then positive, then 0, then negative again. The actual function values in graph II follow the same pattern.
 (b)' = IV, since from left to right, the slopes of the tangents to graph (b) start out at a fixed positive quantity, then suddenly become negative, then positive again. The discontinuities in graph IV indicate sudden changes in the slopes of the tangents.
 (c)' = I, since the slopes of the tangents to graph (c) are negative for $x < 0$ and positive for $x > 0$, as are the function values of graph I.
 (d)' = III, since from left to right, the slopes of the tangents to graph (d) are positive, then 0, then negative, then 0, then positive, then 0, then negative again, and the function values in graph III follow the same pattern.

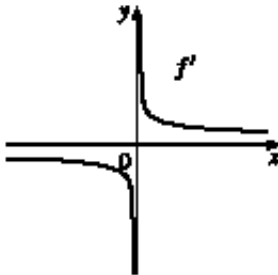
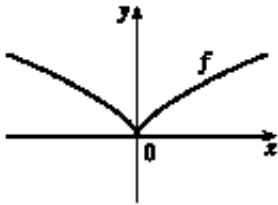
Hints for Exercises 4–11: First plot x -intercepts on the graph of f' for any horizontal tangents on the graph of f . Look for any corners on the graph of f —there will be a discontinuity on the graph of f' . On any interval where f has a tangent with positive (or negative) slope, the graph of f' will be positive (or negative). If the graph of the function is linear, the graph of f' will be a horizontal line.

4.



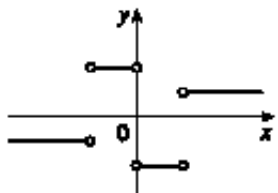
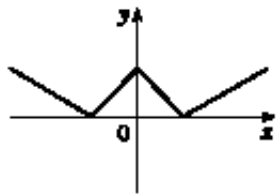
Hints for Exercises 4–11: First plot x -intercepts on the graph of f' for any horizontal tangents on the graph of f . Look for any corners on the graph of f —there will be a discontinuity on the graph of f' . On any interval where f has a tangent with positive (or negative) slope, the graph of f' will be positive (or negative). If the graph of the function is linear, the graph of f' will be a horizontal line.

8.

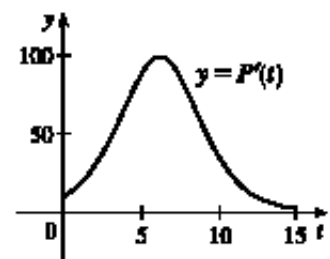


Hints for Exercises 4–11: First plot x -intercepts on the graph of f' for any horizontal tangents on the graph of f . Look for any corners on the graph of f —there will be a discontinuity on the graph of f' . On any interval where f has a tangent with positive (or negative) slope, the graph of f' will be positive (or negative). If the graph of the function is linear, the graph of f' will be a horizontal line.

9.



12. The slopes of the tangent lines on the graph of $y = P(t)$ are always positive, so the y -values of $y = P'(t)$ are always positive. These values start out relatively small and keep increasing, reaching a maximum at about $t = 6$. Then the y -values of $y = P'(t)$ decrease and get close to zero. The graph of P' tells us that the yeast culture grows most rapidly after 6 hours and then the growth rate declines.



$$\begin{aligned} 19. f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{\left[\frac{1}{2}(x+h) - \frac{1}{3}\right] - \left(\frac{1}{2}x - \frac{1}{3}\right)}{h} = \lim_{h \rightarrow 0} \frac{\frac{1}{2}x + \frac{1}{2}h - \frac{1}{3} - \frac{1}{2}x + \frac{1}{3}}{h} \\ &= \lim_{h \rightarrow 0} \frac{\frac{1}{2}h}{h} = \lim_{h \rightarrow 0} \frac{1}{2} = \frac{1}{2} \end{aligned}$$

Domain of f = domain of $f' = \mathbb{R}$.

$$\begin{aligned} 23. g'(x) &= \lim_{h \rightarrow 0} \frac{g(x+h) - g(x)}{h} = \lim_{h \rightarrow 0} \frac{\sqrt{1+2(x+h)} - \sqrt{1+2x}}{h} \left[\frac{\sqrt{1+2(x+h)} + \sqrt{1+2x}}{\sqrt{1+2(x+h)} + \sqrt{1+2x}} \right] \\ &= \lim_{h \rightarrow 0} \frac{(1+2x+2h) - (1+2x)}{h \left[\sqrt{1+2(x+h)} + \sqrt{1+2x} \right]} = \lim_{h \rightarrow 0} \frac{2}{\sqrt{1+2x+2h} + \sqrt{1+2x}} = \frac{2}{2\sqrt{1+2x}} = \frac{1}{\sqrt{1+2x}} \end{aligned}$$

Domain of $g = \left[-\frac{1}{2}, \infty\right)$, domain of $g' = \left(-\frac{1}{2}, \infty\right)$.

30. (a) $P'(t)$ is the rate at which the percentage of Americans under the age of 18 is changing with respect to time. Its units are percent per year (%/yr).

(b) To find $P'(t)$, we use $\lim_{h \rightarrow 0} \frac{P(t+h) - P(t)}{h} \approx \frac{P(t+h) - P(t)}{h}$ for small values of h .

$$\text{For 1950: } P'(1950) \approx \frac{P(1960) - P(1950)}{1960 - 1950} = \frac{35.7 - 31.1}{10} = 0.46$$

For 1960: We estimate $P'(1960)$ by using $h = -10$ and $h = 10$, and then average the two results to obtain a final estimate.

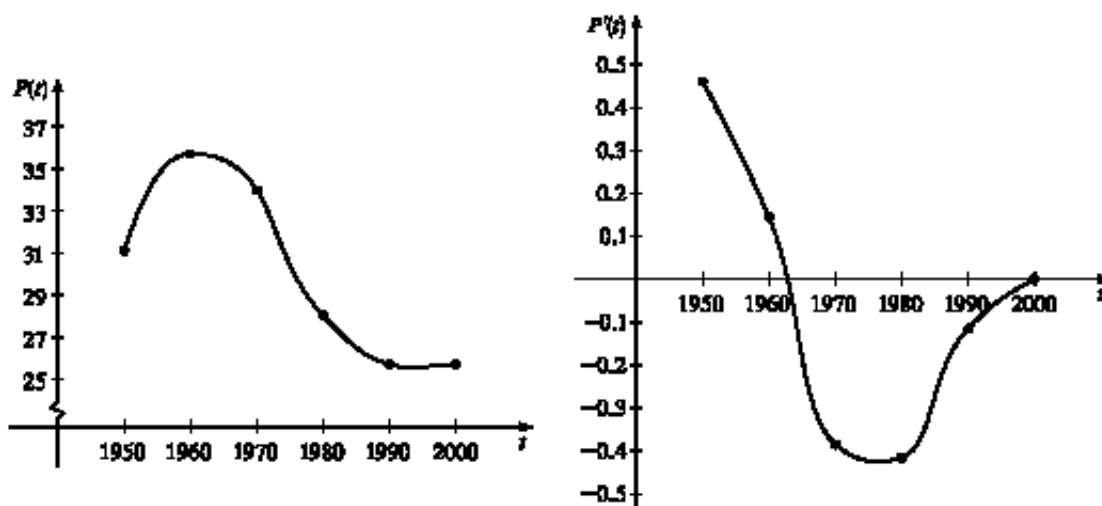
$$h = -10 \Rightarrow P'(1960) \approx \frac{P(1950) - P(1960)}{1950 - 1960} = \frac{31.1 - 35.7}{-10} = 0.46$$

$$h = 10 \Rightarrow P'(1960) \approx \frac{P(1970) - P(1960)}{1970 - 1960} = \frac{34.0 - 35.7}{10} = -0.17$$

So we estimate that $P'(1960) \approx \frac{1}{2}[0.46 + (-0.17)] = 0.145$.

t	1950	1960	1970	1980	1990	2000
$P'(t)$	0.460	0.145	-0.385	-0.415	-0.115	0.000

(c)



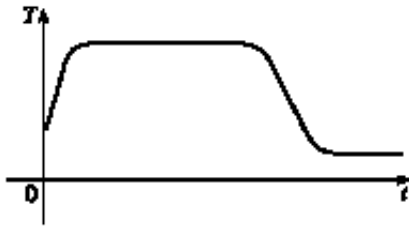
(d) We could get more accurate values for $P'(t)$ by obtaining data for the mid-decade years 1955, 1965, 1975, 1985, and 1995.

31. f is not differentiable at $x = -4$, because the graph has a corner there, and at $x = 0$, because there is a discontinuity there.

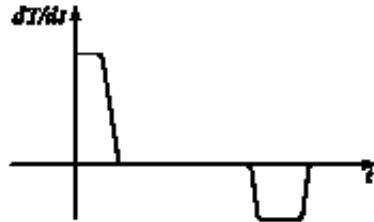
38. Where d has horizontal tangents, only c is 0, so $d' = c$. c has negative tangents for $x < 0$ and b is the only graph that is negative for $x < 0$, so $c' = b$. b has positive tangents on \mathbb{R} (except at $x = 0$), and the only graph that is positive on the same domain is a , so $b' = a$. We conclude that $d = f$, $c = f'$, $b = f''$, and $a = f'''$.

39. We can immediately see that a is the graph of the acceleration function, since at the points where a has a horizontal tangent, neither c nor b is equal to 0. Next, we note that $a = 0$ at the point where b has a horizontal tangent, so b must be the graph of the velocity function, and hence, $b' = a$. We conclude that c is the graph of the position function.

50. (a)



(c)



(b) The initial temperature of the water is close to room temperature because of the water that was in the pipes. When the water from the hot water tank starts coming out, dT/dt is large and positive as T increases to the temperature of the water in the tank. In the next phase, $dT/dt = 0$ as the water comes out at a constant, high temperature. After some time, dT/dt becomes small and negative as the contents of the hot water tank are exhausted. Finally, when the hot water has run out, dT/dt is once again 0 as the water maintains its (cold) temperature.