Name: __________________ Class:___ Date: _____________

1. A function f has an **absolute minimum** at $x = c$ if $f(c)$ is the smallest function value on the entire domain of f, whereas f has a **local minimum** at c if $f(c)$ is the smallest function value when x is near c.

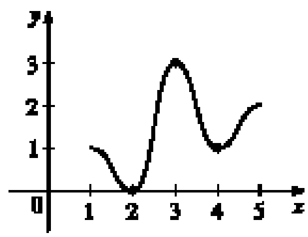
2. (a) The Extreme Value Theorem

(b) See the Closed Interval Method.

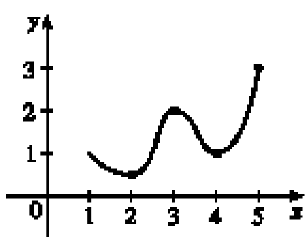
3. Absolute maximum at b; absolute minimum at d; local maxima at b and e; local minima at d and s; neither a maximum nor a minimum at a, c, r, and t.

6. Absolute maximum value is $f(7) = 5$; absolute minimum value is $f(1) = 0$; local maximum values are $f(0) = 2$, $f(3) = 4$, and $f(5) = 3$; local minimum values are $f(1) = 0$, $f(4) = 2$, and $f(6) = 1$.

7. Absolute minimum at 2, absolute maximum at 3, local minimum at 4



9. Absolute maximum at 5, absolute minimum at 2, local maximum at 3, local minima at 2 and 4



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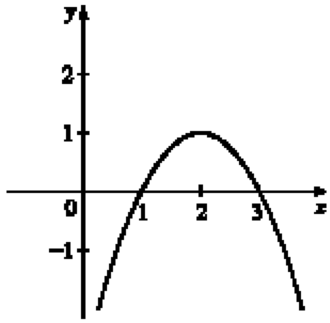
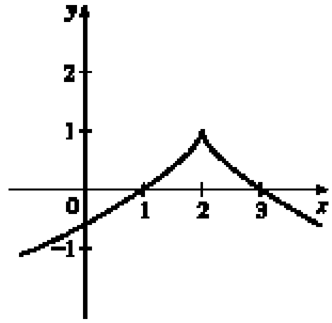
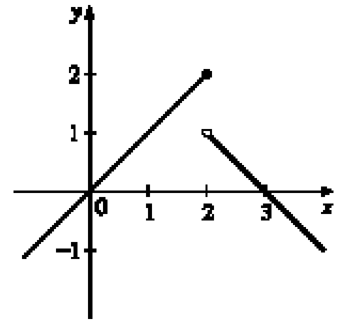
11. (a)



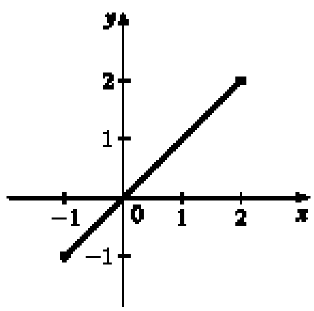
(b)



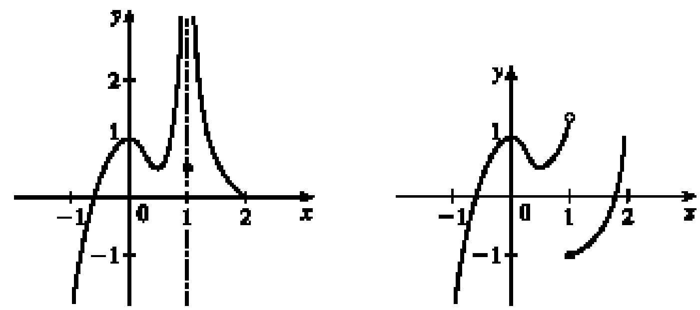
(c)



12. (a) Note that a local maximum cannot occur at an endpoint.



(b)



Note: By the Extreme Value Theorem, f must *not* be continuous.

30. $h(p) = \dfrac{p-1}{p^2+4} \quad \Rightarrow \quad h'(p) = \dfrac{(p^2+4)(1)-(p-1)(2p)}{(p^2+4)^2} = \dfrac{p^2+4-2p^2+2p}{(p^2+4)^2} = \dfrac{-p^2+2p+4}{(p^2+4)^2}$.

$h'(p) = 0 \quad \Rightarrow \quad p = \dfrac{-2 \pm \sqrt{4+16}}{-2} = 1 \pm \sqrt{5}$. The critical numbers are $1 \pm \sqrt{5}$. [$h'(p)$ exists for all real numbers.]

36. $f(x) = xe^{2x} \quad \Rightarrow \quad f'(x) = x(2e^{2x}) + e^{2x} = e^{2x}(2x+1)$. Since e^{2x} is never 0, we have $f'(x) = 0$ only when $2x + 1 = 0 \quad \Leftrightarrow \quad x = -\frac{1}{2}$. So $-\frac{1}{2}$ is the only critical number.

39. $f(x) = 2x^3 - 3x^2 - 12x + 1$, $[-2, 3]$. $f'(x) = 6x^2 - 6x - 12 = 6(x^2 - x - 2) = 6(x-2)(x+1) = 0 \quad \Leftrightarrow$ $x = 2, -1$. $f(-2) = -3$, $f(-1) = 8$, $f(2) = -19$, and $f(3) = -8$. So $f(-1) = 8$ is the absolute maximum value and $f(2) = -19$ is the absolute minimum value.

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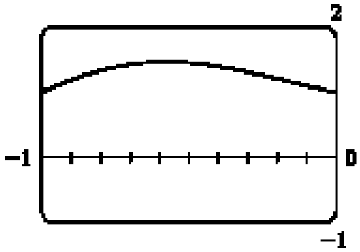
52. (a)



From the graph, it appears that the absolute maximum value is about $f(-0.58) = 1.47$, and the absolute minimum value is about $f(-1) = f(0) = 1.00$; that is, at both endpoints.

(b) $f(x) = e^{x^3 - x} \quad \Rightarrow \quad f'(x) = e^{x^3 - x}(3x^2 - 1)$. So $f'(x) = 0$ on $[-1, 0] \quad \Rightarrow \quad x = -\sqrt{1/3}$.

$f(-1) = f(0) = 1$ (minima) and $f\left(-\sqrt{1/3}\right) = e^{-\sqrt{3}/9 + \sqrt{3}/3} = e^{2\sqrt{3}/9}$ (maximum).

55. The density is defined as $\rho = \dfrac{\text{mass}}{\text{volume}} = \dfrac{1000}{V(T)}$ (in g/cm^3). But a critical point of ρ will also be a critical point of V $\left[\text{since } \dfrac{d\rho}{dT} = -1000V^{-2}\dfrac{dV}{dT} \text{ and } V \text{ is never } 0\right]$, and V is easier to differentiate than ρ.

$V(T) = 999.87 - 0.06426T + 0.0085043T^2 - 0.0000679T^3 \quad \Rightarrow \quad V'(T) = -0.06426 + 0.0170086T - 0.0002037T^2$.

Setting this equal to 0 and using the quadratic formula to find T, we get

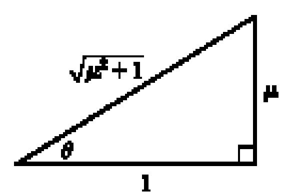
$T = \dfrac{-0.0170086 \pm \sqrt{0.0170086^2 - 4 \cdot 0.0002037 \cdot 0.06426}}{2(-0.0002037)} \approx 3.9665°\text{C}$ or $79.5318°\text{C}$. Since we are only interested in the region $0°\text{C} \le T \le 30°\text{C}$, we check the density ρ at the endpoints and at $3.9665°\text{C}$: $\rho(0) \approx \dfrac{1000}{999.87} \approx 1.00013$;

$\rho(30) \approx \dfrac{1000}{1003.7628} \approx 0.99625$; $\rho(3.9665) \approx \dfrac{1000}{999.7447} \approx 1.000255$. So water has its maximum density at about $3.9665°\text{C}$.

56. $F = \dfrac{\mu W}{\mu \sin\theta + \cos\theta} \quad \Rightarrow \quad \dfrac{dF}{d\theta} = \dfrac{(\mu \sin\theta + \cos\theta)(0) - \mu W(\mu\cos\theta - \sin\theta)}{(\mu\sin\theta + \cos\theta)^2} = \dfrac{-\mu W(\mu\cos\theta - \sin\theta)}{(\mu\sin\theta + \cos\theta)^2}$.

So $\dfrac{dF}{d\theta} = 0 \quad \Rightarrow \quad \mu\cos\theta - \sin\theta = 0 \quad \Rightarrow \quad \mu = \dfrac{\sin\theta}{\cos\theta} = \tan\theta$. Substituting $\tan\theta$ for μ in F gives us

$F = \dfrac{(\tan\theta)W}{(\tan\theta)\sin\theta + \cos\theta} = \dfrac{W\tan\theta}{\dfrac{\sin^2\theta}{\cos\theta} + \cos\theta} = \dfrac{W\tan\theta\cos\theta}{\sin^2\theta + \cos^2\theta} = \dfrac{W\sin\theta}{1} = W\sin\theta$.



If $\tan\theta = \mu$, then $\sin\theta = \dfrac{\mu}{\sqrt{\mu^2 + 1}}$ (see the figure), so $F = \dfrac{\mu}{\sqrt{\mu^2 + 1}}W$.

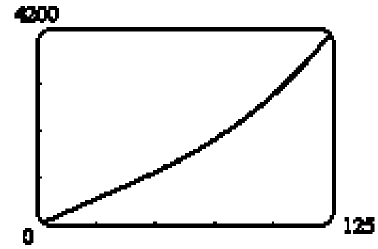
We compare this with the value of F at the endpoints: $F(0) = \mu W$ and $F\left(\frac{\pi}{2}\right) = W$.

Now because $\dfrac{\mu}{\sqrt{\mu^2 + 1}} \le 1$ and $\dfrac{\mu}{\sqrt{\mu^2 + 1}} \le \mu$, we have that $\dfrac{\mu}{\sqrt{\mu^2 + 1}}W$ is less than or equal to each of $F(0)$ and $F\left(\frac{\pi}{2}\right)$.

Hence, $\dfrac{\mu}{\sqrt{\mu^2 + 1}}W$ is the absolute minimum value of $F(\theta)$, and it occurs when $\tan\theta = \mu$.

58. (a) The equation of the graph in the figure is

$$v(t) = 0.00146t^3 - 0.11553t^2 + 24.98169t - 21.26872.$$



(b) $a(t) = v'(t) = 0.00438t^2 - 0.23106t + 24.98169 \quad \Rightarrow$

$a'(t) = 0.00876t - 0.23106. \quad a'(t) = 0 \quad \Rightarrow \quad t_1 = \frac{0.23106}{0.00876} \approx 26.4.$

$a(0) \approx 24.98$, $a(t_1) \approx 21.93$, and $a(125) \approx 64.54$.

The maximum acceleration is about $64.5\ \text{ft/s}^2$ and the minimum acceleration is about $21.93\ \text{ft/s}^2$.

59. (a) $v(r) = k(r_0 - r)r^2 = kr_0r^2 - kr^3 \quad \Rightarrow \quad v'(r) = 2kr_0r - 3kr^2. \quad v'(r) = 0 \quad \Rightarrow \quad kr(2r_0 - 3r) = 0 \quad \Rightarrow$ $r = 0$ or $\frac{2}{3}r_0$ (but 0 is not in the interval). Evaluating v at $\frac{1}{2}r_0$, $\frac{2}{3}r_0$, and r_0, we get $v\left(\frac{1}{2}r_0\right) = \frac{1}{8}kr_0^3$, $v\left(\frac{2}{3}r_0\right) = \frac{4}{27}kr_0^3$, and $v(r_0) = 0$. Since $\frac{4}{27} > \frac{1}{8}$, v attains its maximum value at $r = \frac{2}{3}r_0$. This supports the statement in the text.

(b) From part (a), the maximum value of v is $\frac{4}{27}kr_0^3$.

(c)