Math 1a. Introduction to Calculus
Review Guide for the Final Exam

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Fall 2005

Final Exam Details

The final exam is comprehensive and will cover the entire course; however, about 30–40% of the problems will be about the material covered in Chapter 5. The exam will be on Saturday, January 14, 2006 at 9:15 AM to 12:15 PM. Students whose last name begins with A-L will take the exam in Science Center C. Students whose name begins will M-Z will take the exam in Science Center B. If you have a conflict with the final exam, the Registrar will contact you. Only the Final Exams Office can schedule out-of-sequence final exams.

Review Sessions

There will be three course-wide review sessions. Each review session will cover a different part of the course.

- Review Session I—Friday, January 6 at 2-4 PM in Science C.
- Review Session II—Monday, January 9 at 3-5 PM in Science D.
- Review Session III—Wednesday, January 11 at 3-5 PM in Science D

We plan to videotape each review session. You will be able to access the videos by clicking on Lecture Videos at the course website.

Office Hours and the MQC

Office Hours for TFs and CAs are posted at

`http://www.courses.fas.harvard.edu/~math1a/exams/`

The Math Question Center will be in operation from Sunday, January 8 though Thursday, January 12 in Loker Commons at 8–10 PM.

Studying and Reviewing

- You can find copies of old midterms and final exams as well as solutions by clicking on Previous Exams at the course website.
- You should also try working some of the problems in the review sections of Chapters 2–5. We will post solutions to these problems on the course website on the Exams page.
- Be sure to take advantage of the Review Sessions, TF office hours, CA sections, and the MQC in Loker Commons.

Topics for the Final Exam

- To understand how the tangent line as a limit of secant lines (Section 2.1).
- To understand the concepts of average and instantaneous velocity, described in numerical, graphical, and physical terms (Section 2.1).
- To understand the concept of local linearity (Section 2.1).
- To be able to approximate the slope of a tangent line using the slopes of secant lines (Section 2.1).
- To understand the idea of a limit (both finite and infinite) from descriptive, graphical, and numerical points of view.[1] (Section 2.2).
- To understand and be able to calculate one-sided limits (Section 2.2).
- To understand the disadvantages and advantages of trying to find limits numerically and graphically and why guessing the limit does not always work (Section 2.2).

[1] We will not emphasize the ϵ-δ definition of a limit.

- To understand that $\lim_{x\to a} f(x)$ may not be equal to $f(a)$ (Section 2.2). .
- To understand and be able to compute limits algebraically (Section 2.3). .
- To understand and be able to evaluate limits from a graphical point of view (Section 2.3).
- To know and be familiar with examples where limits do not exist (Section 2.3).
- To be able to compute limits when the limit laws do not apply such as in the case of the Squeeze Theorem (Section 2.3).
- To understand the geometric and mathematical definitions of continuity (Section 2.4).
- To understand discontinuity and be familiar with different examples of discontinuous functions (Section 2.4).
- To understand and be able to apply the Intermediate Value Theorem (Section 2.4).
- To understand and be able to apply the geometric and limit definitions of asymptotes, particularly as they pertain to rational functions (Section 2.5).
- To understand and be able to compute infinite limits (Section 2.5).
- To understand the idea that the slope of a tangent line is the limit of the slope of secant lines and to be able to compute such limits (Section 2.6).
- To understand and be able to compute the instantaneous rate of change as the limit of average rates of change (Section 2.6).
- To understand the definition of the derivative at $x = a$,

  $$f'(a) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h} = \lim_{x\to a} \frac{f(x) - f(a)}{x - a}.$$

  (Section 2.7).
- To understand and be able to compute the equation of a tangent line with f' notation (Section 2.7).
- To understand and be able to apply the derivative as an approximate rate of change when working with discrete data (Section 2.7).
- To understand and be able to use the units associated with $f'(x)$ (Section 2.7).
- To understand the concept of a differentiable function, interpreted graphically, algebraically, and descriptively (Section 2.8).
- To understand and be able to obtain the derivative function f' by first considering the derivative at a point x, and then treating x as a variable (Section 2.8).

- To understand how a function can fail to be differentiable (Section 2.8).
- To understand and be able to sketch the derivative function from the graph of the original function (Section 2.9).
- To understand and be able use the first derivative to determine if a function is increasing or decreasing (Section 2.9).
- To understand and be able use the second derivative to determine concavity and points of inflection (Section 2.9).
- To understand the geometric description of concavity and the relationship between concavity and the increasing/decreasing behavior of the first derivative (Section 2.9).
- To understand and be able use the power rule and the derivative of e^x. These rules should be developed from the definition of the derivative (Section 3.1).
- To understand and be able to apply the definition of e (Section 3.1).
- To understand and be able the product and quotient rules for differentiating functions (Section 3.2).
- To understand and be able to applying the concept of a derivative to applications in the natural and social sciences (Section 3.3).
- To understand and be able to evaluate the derivatives of trigonometric functions (Section 3.4).
- To understand and be able to apply the chain rule when differentiating composite functions (Section 3.5).
- To understand and be able to apply the chain rule when differentiating implicitly defined functions (Section 3.6).
- To understand and be able to apply the derivatives of the inverse trigonometric functions (Section 3.6).
- To understand and be able to determine when two curves are orthogonal (Section 3.6).
- To understand and be able to apply the basic logarithmic differentiation formula (Section 3.7).
- To understand and be able to apply the technique of logarithmic differentiation (Section 3.7).
- To understand and be able to apply the concept of e as a limit (Section 3.7).
- To understand and be able to apply the process of linearizing a function at $x = a$ (Section 3.8).

- To understand and be able to the differential as the difference between the linearization of a function and the function itself (Section 3.8).
- To understand and be able to the concept of related rates (Section 4.1).
- To understand the definition of local and absolute extrema both intuitively and precisely (Section 4.2).
- To understand and be able to apply the Extreme Value Theorem and Fermat's Theorem (Section 4.2).
- To understand and be able to find critical values and use the closed interval method (Section 4.2).
- To be able to use the first derivative to determine whether a function is increasing or decreasing (Section 4.3).
- To understand and be able to apply the First and Second Derivative Tests for local maxima and minima (Section 4.3).
- To understand and be able to apply the Mean Value Theorem (Section 4.3).
- To understand the relationship between concavity and the behavior of the first derivative (Section 4.3).
- To be able to use the second derivative to determine concavity and points of inflection (Section 4.3).
- To be able to use calculus to sketch the graphs of functions (Section 4.3).
- To recognize the indeterminate forms $0/0$, ∞/∞, and $\infty - \infty$ and be able to apply L'Hôspital's Rule when evaluating limits of these indeterminate forms (Section 4.5).
- To understand l'Hospital's Rule in terms of relative rates of change (Section 4.5).
- To recognize the indeterminate forms $0 \cdot \infty$, 1^∞, ∞^0, 0^∞, and 0^0 and be able to apply l'Hospital's Rule when evaluating limits of these indeterminate forms (Section 4.5).
- To be able to set up and solve optimization problems using calculus (Section 4.6).
- To be able to apply calculus to problems in business and economics (Section 4.7).
- To be able to use the Newton-Raphson algorithm to approximate roots of functions (Section 4.8).
- To understand and be able to compute various antiderivatives (Section 4.9).
- To understand the relationship between a differential equation and its direction field (Section 4.9).

- To understand and be able to apply calculus to solve problems about rectilinear motion (Section 4.9).
- To understand and be able to find net change (Section 5.1).
- To understand and be able to the concept of the definite integral as net change and as the area under a curve (Section 5.1).
- To understand the concept of signed area (Section 5.1).
- To understand and be able to apply the definition of the definite integral (Section 5.2).
- To be able to evaluate definite integrals using Riemann sums (Section 5.2).
- To understand and be able to apply the properties of the definite integral (Section 5.2).
- To be able to apply the Fundamental Theorem of Calculus, especially Part II—if F is any antiderivative of f, then

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

(Section 5.3).

- To understand the area function (Section 5.4).
- To understand and be able to apply the Fundamental Theorem of Calculus (Section 5.4).
- To be able to evaluate definite and indefinite integrals using substitution (Section 5.5).