Name:______________________________

Math 1a Midterm Examination II—Tuesday, December 13, 2005

Please circle your section:

Bret Benesh	Tatyana Chmutova	Maksym Fedorchuk	Thomas Judson
MWF 9–10	10-11 MWF	10–11 MWF	11–12 MWF

Tatyana Chmutova	Robin Gottlieb	Robin Gottlieb
MWF 12–1	TTh 10–11:30	TTh 11:30–1

Problem Number	Possible Points	Score
1	21	
2	10	
3	9	
4	10	
5	9	
6	8	
7	10	
8	8	
9	9	
10	6	
Total	100	

Directions—Please Read Carefully! You have two hours to take this midterm. Make sure to use correct mathematical notation. Pace yourself by keeping track of how many problems you have left to go and how much time remains. You do not have to answer the problems in any particular order, so move to another problem if you find you are stuck or spending too much time on a single problem. To receive full credit on a problem, you will need to justify your answers carefully—unsubstantiated answers will receive little or no credit (except if the directions for that question specifically say no justification is necessary, such as in a True/False section). Please be sure to write neatly—illegible answers will receive little or no credit. If more space is needed, use the back of the previous page to continue your work. Be sure to make a note of this on the problem page so that the grader knows where to find your answers. *Calculators are not allowed.* **Good Luck!!!**

1. Find the derivative of each of the following functions. ***You do not need to simplify your answers***. (21 points)

   (a) $y = (x^3 + 3x^2 - 5)^{100}$.

   (b) $y = x^5 e^x \cos x$.

(c) $y = \dfrac{e^{2x}}{\sqrt{1+x^2}}$.

(d) $y = \arcsin x + \tan^{-1} x + \tan x$.

(e) $y = x^{\sin x}$.

(f) $y = \ln\left(\dfrac{(x^2-2)^3}{(x^2+3)^5\sqrt{x^2+1}}\right)$.

(g) Use implicit differentiation to find $\frac{dy}{dx}$ if $y^5 + x^2y^3 = 6x^2 - y$.

2. A weather balloon that is rising vertically is being observed from a point on the ground 300 feet from the spot directly beneath the balloon. At what rate is the balloon rising when the angle between the ground and the observer's line of sight $\pi/4$ and is increasing at $\pi/60$ radians per second? (10 points)

3. Using linear approximation, find a good estimate for $\sqrt[10]{0.97}$. (9 points)

4. A manufacturer can produce a pair of earrings at a cost of \$3. The earrings have been for \$5 a pair and, at this price, consumers have been buying 4,000 pairs per month. The manufacturer is planning to raise the price of the earrings and estimates that for each \$1 increase, 400 fewer pairs of earrings will be sold each month. (10 points)

   (a) Find the price function, $p(x)$, where x is the number of units sold and $p(x)$ is the price of one pair of earrings.

   (b) Find the number of earrings the manufacturer should sell to maximize profit. Be sure to prove that your answer is definitely a maximum.

   (c) At what price should the manufacturer sell each pair of earrings to maximize profit?

5. (9 points)

(a) Evaluate $\lim_{x \to 0} \frac{\ln(1+x)}{x}$.

(b) Evaluate $\lim_{x \to 0^+} \frac{\cos(2x)}{x}$.

(c) Suppose that f and f' are continuous functions and $f(0) = 0$. If

$$\lim_{x \to 0} \frac{f(x)}{\sin(2x)} = 3,$$

evaluate $f'(0)$.

6. (8 points)

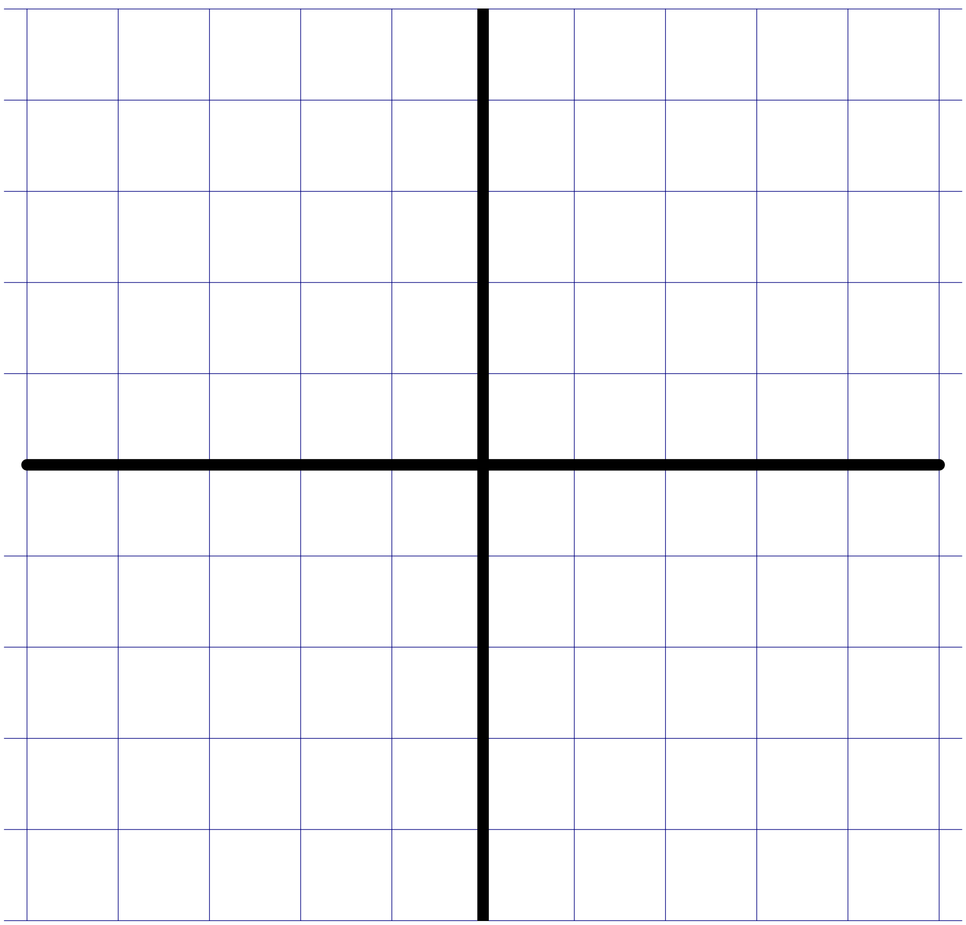
(a) Accurately state the Mean Value Theorem.

(b) Let f be a differentiable function such that $f(0) = 0$ and $f'(x) \leq 1$ for all x. Use the Mean Value Theorem to show that $f(2) \neq 3$.

7. Assume that f is a continuous function on the closed interval $[-5, 5]$ with $f(-5) = -4$ and $f(5) = 1$. Also, assume that f' and f'' exist and are continuous on $(-5, 5)$. Use the information in the table below to sketch a possible graph of f. (10 points)

x	$-5 \leq x < -3$	-3	$-3 < x < -1$	-1	$-1 < x < 1$
$f'(x)$	$+$	0	$+$	$+$	$+$
$f''(x)$	$-$	0	$+$	0	$-$

x	1	$1 < x < 3$	3	$3 < x < 4$	4	$4 < x \leq 5$
$f'(x)$	0	$-$	$-$	$-$	0	$+$
$f''(x)$	$-$	$-$	0	$+$	$+$	$+$



8. Suppose that f and g are differentiable functions with the following table of values for the functions and their derivatives. (8 points)

x	-2	-1	2
$f(x)$	1	-3	-1
$f'(x)$	-3	2	3
$g(x)$	-1	4	2
$g'(x)$	5	-1	-2

(a) If $h(x) = f(x) \cdot g(x)$, evaluate $h'(2)$.

(b) If $h(x) = f(x)/g(x)$, evaluate $h'(-1)$.

(c) If $h(x) = f(g(x))$, evaluate $h'(-2)$.

(d) If $h(x) = f^{-1}(x)$, evaluate $h'(1)$. [***Hint***: Use the fact that $f(h(x)) = x$.]

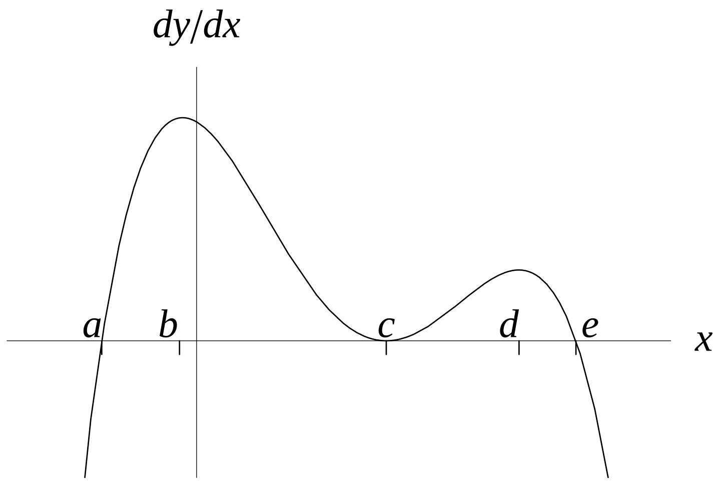
9. Find all local and global maxima and minima of each of the following functions. (9 points)

(a) $f(x) = 2x^3 - 9x^2 + 12x + 1$ on the interval $[0, 3]$.

(b) $f(x) = \dfrac{1}{(x-2)^2}$ on the interval $[0, 3]$.

(c) $f(x) = |x - 2|$ on the interval $[0, 3]$.

10. Below is a graph of the *derivative* of $f(x)$. ***Caution***—this is not $f(x)$; it is $f'(x)$. Use the graph of the derivative to answer the following questions about the function $f(x)$. (6 points)



(a) For what values of x is the graph of f increasing?

(b) For what values of x is the graph of f concave up?

(c) Find and classify the critical points of $f(x)$. Is the critical point a local maximum, a local minimum, or neither?