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1. $y = (x^4 - 3x^2 + 5)^3 \quad \Rightarrow$

$y' = 3(x^4 - 3x^2 + 5)^2 \dfrac{d}{dx}(x^4 - 3x^2 + 5) = 3(x^4 - 3x^2 + 5)^2(4x^3 - 6x) = 6x(x^4 - 3x^2 + 5)^2(2x^2 - 3)$

2. $y = \cos(\tan x) \quad \Rightarrow \quad y' = -\sin(\tan x)\dfrac{d}{dx}(\tan x) = -\sin(\tan x)(\sec^2 x)$

3. $y = \sqrt{x} + \dfrac{1}{\sqrt[3]{x^4}} = x^{1/2} + x^{-4/3} \quad \Rightarrow \quad y' = \frac{1}{2}x^{-1/2} - \frac{4}{3}x^{-7/3} = \dfrac{1}{2\sqrt{x}} - \dfrac{4}{3\sqrt[3]{x^7}}$

4. $y = \dfrac{3x-2}{\sqrt{2x+1}} \quad \Rightarrow$

$y' = \dfrac{\sqrt{2x+1}\,(3) - (3x-2)\frac{1}{2}(2x+1)^{-1/2}(2)}{\left(\sqrt{2x+1}\right)^2} \cdot \dfrac{(2x+1)^{1/2}}{(2x+1)^{1/2}} = \dfrac{3(2x+1)-(3x-2)}{(2x+1)^{3/2}} = \dfrac{3x+5}{(2x+1)^{3/2}}$

5. $y = 2x\sqrt{x^2+1} \quad \Rightarrow$

$y' = 2x \cdot \frac{1}{2}(x^2+1)^{-1/2}(2x) + \sqrt{x^2+1}\,(2) = \dfrac{2x^2}{\sqrt{x^2+1}} + 2\sqrt{x^2+1} = \dfrac{2x^2 + 2(x^2+1)}{\sqrt{x^2+1}} = \dfrac{2(2x^2+1)}{\sqrt{x^2+1}}$

6. $y = \dfrac{e^x}{1+x^2} \quad \Rightarrow \quad y' = \dfrac{(1+x^2)e^x - e^x(2x)}{(1+x^2)^2} = \dfrac{e^x(x^2-2x+1)}{(1+x^2)^2} = \dfrac{e^x(x-1)^2}{(1+x^2)^2}$

7. $y = e^{\sin 2\theta} \quad \Rightarrow \quad y' = e^{\sin 2\theta}\dfrac{d}{d\theta}(\sin 2\theta) = e^{\sin 2\theta}(\cos 2\theta)(2) = 2\cos 2\theta\, e^{\sin 2\theta}$

8. $y = e^{-t}(t^2 - 2t + 2) \quad \Rightarrow \quad y' = e^{-t}(2t-2) + (t^2-2t+2)(-e^{-t}) = e^{-t}(2t - 2 - t^2 + 2t - 2) = e^{-t}(-t^2 + 4t - 4)$

9. $y = \dfrac{t}{1-t^2} \quad \Rightarrow \quad y' = \dfrac{(1-t^2)(1) - t(-2t)}{(1-t^2)^2} = \dfrac{1-t^2+2t^2}{(1-t^2)^2} = \dfrac{t^2+1}{(1-t^2)^2}$

10. $y = \sin^{-1}(e^x) \quad \Rightarrow \quad y' = \dfrac{1}{\sqrt{1-(e^x)^2}} \cdot e^x = e^x/\sqrt{1-e^{2x}}$

11. $y = xe^{-1/x} \quad \Rightarrow \quad y' = xe^{-1/x}(1/x^2) + e^{-1/x} \cdot 1 = e^{-1/x}(1/x + 1)$

12. $y = x^r e^{sx} \quad \Rightarrow \quad y' = x^r(se^{sx}) + e^{sx}(rx^{r-1}) = e^{sx}x^{r-1}(sx + r)$

13. $\frac{d}{dx}(xy^4 + x^2y) = \frac{d}{dx}(x+3y) \Rightarrow x \cdot 4y^3y' + y^4 \cdot 1 + x^2 \cdot y' + y \cdot 2x = 1 + 3y' \Rightarrow$

$y'(4xy^3 + x^2 - 3) = 1 - y^4 - 2xy \Rightarrow y' = \dfrac{1 - y^4 - 2xy}{4xy^3 + x^2 - 3}$

14. $y = \ln(\csc 5x) \Rightarrow y' = \dfrac{1}{\csc 5x}(-\csc 5x \cot 5x)(5) = -5\cot 5x$

15. $y = \dfrac{\sec 2\theta}{1 + \tan 2\theta} \Rightarrow$

$y' = \dfrac{(1+\tan 2\theta)(\sec 2\theta \tan 2\theta \cdot 2) - (\sec 2\theta)(\sec^2 2\theta \cdot 2)}{(1+\tan 2\theta)^2} = \dfrac{2\sec 2\theta\,[(1+\tan 2\theta)\tan 2\theta - \sec^2 2\theta]}{(1+\tan 2\theta)^2}$

$= \dfrac{2\sec 2\theta\,(\tan 2\theta + \tan^2 2\theta - \sec^2 2\theta)}{(1+\tan 2\theta)^2} = \dfrac{2\sec 2\theta\,(\tan 2\theta - 1)}{(1+\tan 2\theta)^2} \quad [1 + \tan^2 x = \sec^2 x]$

16. $\frac{d}{dx}(x^2\cos y + \sin 2y) = \frac{d}{dx}(xy) \Rightarrow x^2(-\sin y \cdot y') + (\cos y)(2x) + \cos 2y \cdot 2y' = x \cdot y' + y \cdot 1 \Rightarrow$

$y'(-x^2\sin y + 2\cos 2y - x) = y - 2x\cos y \Rightarrow y' = \dfrac{y - 2x\cos y}{2\cos 2y - x^2\sin y - x}$

17. $y = e^{cx}(c\sin x - \cos x) \Rightarrow$

$y' = e^{cx}(c\cos x + \sin x) + ce^{cx}(c\sin x - \cos x)$

$= e^{cx}(c^2\sin x - c\cos x + c\cos x + \sin x) = e^{cx}(c^2\sin x + \sin x) = e^{cx}\sin x\,(c^2+1)$

18. $y = \ln(x^2e^x) = \ln x^2 + \ln e^x = 2\ln x + x \Rightarrow y' = 2/x + 1$

19. $y = \log_5(1+2x) \Rightarrow y' = \dfrac{1}{(1+2x)\ln 5}\dfrac{d}{dx}(1+2x) = \dfrac{2}{(1+2x)\ln 5}$

20. $y = (\ln x)^{\cos x} \Rightarrow \ln y = \cos x\,\ln(\ln x) \Rightarrow \dfrac{y'}{y} = \cos x \cdot \dfrac{1}{\ln x} \cdot \dfrac{1}{x} + (\ln\ln x)(-\sin x) \Rightarrow$

$y' = (\ln x)^{\cos x}\left(\dfrac{\cos x}{x\ln x} - \sin x\,\ln\ln x\right)$

21. $\sin(xy) = x^2 - y \Rightarrow \cos(xy)(xy' + y \cdot 1) = 2x - y' \Rightarrow x\cos(xy)y' + y' = 2x - y\cos(xy) \Rightarrow$

$y'[x\cos(xy) + 1] = 2x - y\cos(xy) \Rightarrow y' = \dfrac{2x - y\cos(xy)}{x\cos(xy) + 1}$

22. $y = \sqrt{t\ln(t^4)} \quad \Rightarrow$

$$y' = \tfrac{1}{2}[t\ln(t^4)]^{-1/2}\,\frac{d}{dt}\,[t\ln(t^4)] = \frac{1}{2\sqrt{t\ln(t^4)}}\cdot\left[1\cdot\ln(t^4) + t\cdot\frac{1}{t^4}\cdot 4t^3\right] = \frac{1}{2\sqrt{t\ln(t^4)}}\cdot[\ln(t^4)+4] = \frac{\ln(t^4)+4}{2\sqrt{t\ln(t^4)}}$$

Or: Since y is only defined for $t > 0$, we can write $y = \sqrt{t\cdot 4\ln t} = 2\sqrt{t\ln t}$. Then

$y' = 2\cdot\dfrac{1}{2\sqrt{t\ln t}}\cdot\left(1\cdot\ln t + t\cdot\dfrac{1}{t}\right) = \dfrac{\ln t+1}{\sqrt{t\ln t}}$. This agrees with our first answer since

$$\frac{\ln(t^4)+4}{2\sqrt{t\ln(t^4)}} = \frac{4\ln t+4}{2\sqrt{t\cdot 4\ln t}} = \frac{4(\ln t+1)}{2\cdot 2\sqrt{t\ln t}} = \frac{\ln t+1}{\sqrt{t\ln t}}.$$

23. $y = 3^{x\ln x} \quad \Rightarrow \quad y' = 3^{x\ln x}\cdot\ln 3\cdot\dfrac{d}{dx}(x\ln x) = 3^{x\ln x}\cdot\ln 3\left(x\cdot\dfrac{1}{x} + \ln x\cdot 1\right) = 3^{x\ln x}\cdot\ln 3(1+\ln x)$

24. $xe^y = y - 1 \quad \Rightarrow \quad xe^y y' + e^y = y' \quad \Rightarrow \quad e^y = y' - xe^y y' \quad \Rightarrow \quad y' = e^y/(1 - xe^y)$

25. $y = \ln\sin x - \tfrac{1}{2}\sin^2 x \quad \Rightarrow \quad y' = \dfrac{1}{\sin x}\cdot\cos x - \tfrac{1}{2}\cdot 2\sin x\cdot\cos x = \cot x - \sin x\cos x$

26. $y = \dfrac{(x^2+1)^4}{(2x+1)^3(3x-1)^5} \quad \Rightarrow$

$$\ln y = \ln\frac{(x^2+1)^4}{(2x+1)^3(3x-1)^5} = \ln(x^2+1)^4 - \ln[(2x+1)^3(3x-1)^5]$$

$$= 4\ln(x^2+1) - [\ln(2x+1)^3 + \ln(3x-1)^5] = 4\ln(x^2+1) - 3\ln(2x+1) - 5\ln(3x-1) \quad \Rightarrow$$

$$\frac{y'}{y} = 4\cdot\frac{1}{x^2+1}\cdot 2x - 3\cdot\frac{1}{2x+1}\cdot 2 - 5\cdot\frac{1}{3x-1}\cdot 3 \quad \Rightarrow \quad y' = \frac{(x^2+1)^4}{(2x+1)^3(3x-1)^5}\left(\frac{8x}{x^2+1} - \frac{6}{2x+1} - \frac{15}{3x-1}\right).$$

[The answer could be simplified to $y' = -\dfrac{(x^2+56x+9)(x^2+1)^3}{(2x+1)^4(3x-1)^6}$, but this is unnecessary.]

27. $y = x\tan^{-1}(4x) \quad \Rightarrow \quad y' = x\cdot\dfrac{1}{1+(4x)^2}\cdot 4 + \tan^{-1}(4x)\cdot 1 = \dfrac{4x}{1+16x^2} + \tan^{-1}(4x)$

28. $y = e^{\cos x} + \cos(e^x) \quad \Rightarrow \quad y' = e^{\cos x}(-\sin x) + [-\sin(e^x)\cdot e^x] = -\sin x\, e^{\cos x} - e^x\sin(e^x)$

29. $y = \ln|\sec 5x + \tan 5x| \quad \Rightarrow$

$$y' = \frac{1}{\sec 5x + \tan 5x}(\sec 5x\,\tan 5x\cdot 5 + \sec^2 5x\cdot 5) = \frac{5\sec 5x\,(\tan 5x + \sec 5x)}{\sec 5x + \tan 5x} = 5\sec 5x$$

30. $y = 10^{\tan\pi\theta} \quad \Rightarrow \quad y' = 10^{\tan\pi\theta}\cdot\ln 10\cdot\sec^2\pi\theta\cdot\pi = \pi(\ln 10)10^{\tan\pi\theta}\sec^2\pi\theta$

31. $y = \tan^2(\sin\theta) = [\tan(\sin\theta)]^2 \quad\Rightarrow\quad y' = 2[\tan(\sin\theta)]\cdot\sec^2(\sin\theta)\cdot\cos\theta$

32. $y = \ln\left|\dfrac{x^2-4}{2x+5}\right| = \ln|x^2-4| - \ln|2x+5| \quad\Rightarrow\quad y' = \dfrac{2x}{x^2-4} - \dfrac{2}{2x+5}$ or $\dfrac{2(x+1)(x+4)}{(x+2)(x-2)(2x+5)}$

33. $y = \sin\left(\tan\sqrt{1+x^3}\right) \quad\Rightarrow\quad y' = \cos\left(\tan\sqrt{1+x^3}\right)\left(\sec^2\sqrt{1+x^3}\right)\left[3x^2/\left(2\sqrt{1+x^3}\right)\right]$

34. $y = \arctan(\arcsin\sqrt{x}) \quad\Rightarrow\quad y' = \dfrac{1}{1+(\arcsin\sqrt{x})^2}\cdot\dfrac{1}{\sqrt{1-x}}\cdot\dfrac{1}{2\sqrt{x}}$

35. $f(t) = \sqrt{4t+1} \quad\Rightarrow\quad f'(t) = \frac{1}{2}(4t+1)^{-1/2}\cdot 4 = 2(4t+1)^{-1/2} \quad\Rightarrow$
$f''(t) = 2(-\frac{1}{2})(4t+1)^{-3/2}\cdot 4 = -4/(4t+1)^{3/2}$, so $f''(2) = -4/9^{3/2} = -\frac{4}{27}$.

36. $g(\theta) = \theta\sin\theta \quad\Rightarrow\quad g'(\theta) = \theta\cos\theta + \sin\theta\cdot 1 \quad\Rightarrow\quad g''(\theta) = \theta(-\sin\theta) + \cos\theta\cdot 1 + \cos\theta = 2\cos\theta - \theta\sin\theta$,
so $g''(\pi/6) = 2\cos(\pi/6) - (\pi/6)\sin(\pi/6) = 2\left(\sqrt{3}/2\right) - (\pi/6)(1/2) = \sqrt{3} - \pi/12$.

37. $f(x) = 2^x \quad\Rightarrow\quad f'(x) = 2^x\ln 2 \quad\Rightarrow\quad f''(x) = (2^x\ln 2)\ln 2 = 2^x(\ln 2)^2 \quad\Rightarrow$
$f'''(x) = (2^x\ln 2)(\ln 2)^2 = 2^x(\ln 2)^3 \quad\Rightarrow\quad \cdots \quad\Rightarrow\quad f^{(n)}(x) = (2^x\ln 2)(\ln 2)^{n-1} = 2^x(\ln 2)^n$

38. $x^6 + y^6 = 1 \quad\Rightarrow\quad 6x^5 + 6y^5y' = 0 \quad\Rightarrow\quad y' = -x^5/y^5 \quad\Rightarrow$

$$y'' = -\frac{y^5(5x^4) - x^5(5y^4y')}{(y^5)^2} = -\frac{5x^4y^4[y - x(-x^5/y^5)]}{y^{10}} = -\frac{5x^4[(y^6+x^6)/y^5]}{y^6} = -\frac{5x^4}{y^{11}}$$

39. $y = 4\sin^2 x \quad\Rightarrow\quad y' = 4\cdot 2\sin x\cos x$. At $\left(\frac{\pi}{6}, 1\right)$, $y' = 8\cdot\frac{1}{2}\cdot\frac{\sqrt{3}}{2} = 2\sqrt{3}$, so an equation of the tangent line is $y - 1 = 2\sqrt{3}\left(x - \frac{\pi}{6}\right)$, or $y = 2\sqrt{3}\,x + 1 - \pi\sqrt{3}/3$.

40. $y = \dfrac{x^2-1}{x^2+1} \quad\Rightarrow\quad y' = \dfrac{(x^2+1)(2x) - (x^2-1)(2x)}{(x^2+1)^2} = \dfrac{4x}{(x^2+1)^2}$.

At $(0,-1)$, $y' = 0$, so an equation of the tangent line is $y + 1 = 0(x-0)$, or $y = -1$.

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41. $y = (2+x)e^{-x} \quad \Rightarrow \quad y' = (2+x)(-e^{-x}) + e^{-x} \cdot 1 = e^{-x}[-(2+x)+1] = e^{-x}(-x-1).$

At $(0, 2)$, $y' = 1(-1) = -1$, so an equation of the tangent line is $y - 2 = -1(x - 0)$, or $y = -x + 2$.

The slope of the normal line is 1, so an equation of the normal line is $y - 2 = 1(x - 0)$, or $y = x + 2$.

42. $x^2 + 4xy + y^2 = 13 \quad \Rightarrow \quad 2x + 4(xy' + y \cdot 1) + 2yy' = 0 \quad \Rightarrow \quad x + 2xy' + 2y + yy' = 0 \quad \Rightarrow$

$2xy' + yy' = -x - 2y \quad \Rightarrow \quad y'(2x + y) = -x - 2y \quad \Rightarrow \quad y' = \dfrac{-x - 2y}{2x + y}.$

At $(2, 1)$, $y' = \dfrac{-2-2}{4+1} = -\dfrac{4}{5}$, so an equation of the tangent line is $y - 1 = -\frac{4}{5}(x - 2)$, or $y = -\frac{4}{5}x + \frac{13}{5}$.

The slope of the normal line is $\frac{5}{4}$, so an equation of the normal line is $y - 1 = \frac{5}{4}(x - 2)$, or $y = \frac{5}{4}x - \frac{3}{2}$.

43. (a) $f(x) = x\sqrt{5 - x} \quad \Rightarrow$

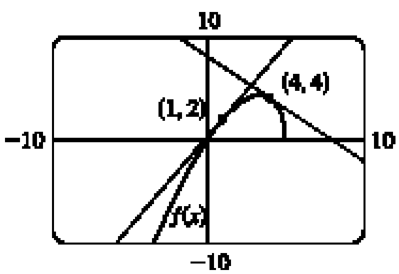
$$f'(x) = x\left[\tfrac{1}{2}(5-x)^{-1/2}(-1)\right] + \sqrt{5-x} = \frac{-x}{2\sqrt{5-x}} + \sqrt{5-x} \cdot \frac{2\sqrt{5-x}}{2\sqrt{5-x}}$$

$$= \frac{-x}{2\sqrt{5-x}} + \frac{2(5-x)}{2\sqrt{5-x}} = \frac{-x + 10 - 2x}{2\sqrt{5-x}} = \frac{10 - 3x}{2\sqrt{5-x}}$$

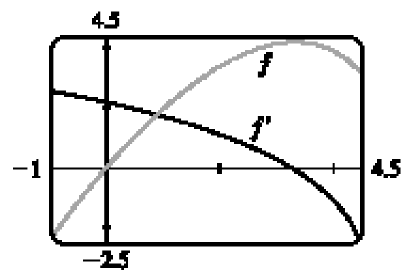
(b) At $(1, 2)$: $f'(1) = \frac{7}{4}$. So an equation of the tangent line is $y - 2 = \frac{7}{4}(x - 1)$ or $y = \frac{7}{4}x + \frac{1}{4}$.

At $(4, 4)$: $f'(4) = -\frac{2}{2} = -1$. So an equation of the tangent line is $y - 4 = -1(x - 4)$ or $y = -x + 8$.

(c)



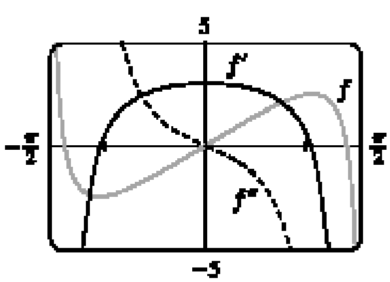
(d)



The graphs look reasonable, since f' is positive where f has tangents with positive slope, and f' is negative where f has tangents with negative slope.

44. (a) $f(x) = 4x - \tan x \quad \Rightarrow \quad f'(x) = 4 - \sec^2 x \quad \Rightarrow \quad f''(x) = -2\sec x\,(\sec x\,\tan x) = -2\sec^2 x\,\tan x.$

(b)



We can see that our answers are reasonable, since the graph of f' is 0 where f has a horizontal tangent, and the graph of f' is positive where f has tangents with positive slope and negative where f has tangents with negative slope. The same correspondence holds between the graphs of f' and f''.

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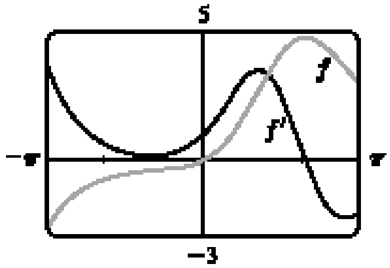
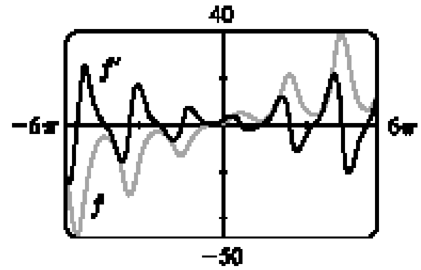
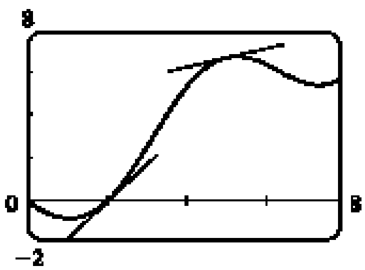
45. $f(x) = xe^{\sin x} \Rightarrow f'(x) = x[e^{\sin x}(\cos x)] + e^{\sin x}(1) = e^{\sin x}(x\cos x + 1)$. As a check on our work, we notice from the graphs that $f'(x) > 0$ when f is increasing. Also, we see in the larger viewing rectangle a certain similarity in the graphs of f and f': the sizes of the oscillations of f and f' are linked.





46. (a)



(b) The average rate of change is larger on $[2, 3]$.

(c) The instantaneous rate of change (the slope of the tangent) is larger at $x = 2$.

(d) $f(x) = x - 2\sin x \Rightarrow f'(x) = 1 - 2\cos x$, so $f'(2) = 1 - 2\cos 2 \approx 1.8323$ and $f'(5) = 1 - 2\cos 5 \approx 0.4327$. So $f'(2) > f'(5)$, as predicted in part (c).

47. (a) $h(x) = f(x)g(x) \Rightarrow h'(x) = f(x)g'(x) + g(x)f'(x) \Rightarrow$
$h'(2) = f(2)g'(2) + g(2)f'(2) = (3)(4) + (5)(-2) = 12 - 10 = 2$

(b) $F(x) = f(g(x)) \Rightarrow F'(x) = f'(g(x))g'(x) \Rightarrow F'(2) = f'(g(2))g'(2) = f'(5)(4) = 11 \cdot 4 = 44$

48. (a) $P(x) = f(x)g(x) \Rightarrow P'(x) = f(x)g'(x) + g(x)f'(x) \Rightarrow$
$P'(2) = f(2)g'(2) + g(2)f'(2) = (1)\left(\frac{6-0}{3-0}\right) + (4)\left(\frac{0-3}{3-0}\right) = (1)(2) + (4)(-1) = 2 - 4 = -2$

(b) $Q(x) = \dfrac{f(x)}{g(x)} \Rightarrow Q'(x) = \dfrac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2} \Rightarrow$
$Q'(2) = \dfrac{g(2)f'(2) - f(2)g'(2)}{[g(2)]^2} = \dfrac{(4)(-1) - (1)(2)}{4^2} = \dfrac{-6}{16} = -\dfrac{3}{8}$

(c) $C(x) = f(g(x)) \Rightarrow C'(x) = f'(g(x))g'(x) \Rightarrow$
$C'(2) = f'(g(2))g'(2) = f'(4)g'(2) = \left(\frac{6-0}{5-3}\right)(2) = (3)(2) = 6$

49. $f(x) = x^2 g(x) \Rightarrow f'(x) = x^2 g'(x) + g(x)(2x) = x[xg'(x) + 2g(x)]$

50. $f(x) = g(x^2) \Rightarrow f'(x) = g'(x^2)(2x) = 2xg'(x^2)$

51. $f(x) = [g(x)]^2 \quad \Rightarrow \quad f'(x) = 2[g(x)]^1 \cdot g'(x) = 2g(x)g'(x)$

52. $f(x) = g(g(x)) \quad \Rightarrow \quad f'(x) = g'(g(x))g'(x)$

53. $f(x) = g(e^x) \quad \Rightarrow \quad f'(x) = g'(e^x)e^x$

54. $f(x) = e^{g(x)} \quad \Rightarrow \quad f'(x) = e^{g(x)}g'(x)$

55. $f(x) = \ln|g(x)| \quad \Rightarrow \quad f'(x) = \dfrac{1}{g(x)}g'(x) = \dfrac{g'(x)}{g(x)}$

56. $f(x) = g(\ln x) \quad \Rightarrow \quad f'(x) = g'(\ln x) \cdot \dfrac{1}{x} = \dfrac{g'(\ln x)}{x}$

57. $h(x) = \dfrac{f(x)g(x)}{f(x) + g(x)} \quad \Rightarrow$

$$h'(x) = \frac{[f(x) + g(x)]\,[f(x)g'(x) + g(x)f'(x)] - f(x)g(x)\,[f'(x) + g'(x)]}{[f(x) + g(x)]^2}$$

$$= \frac{[f(x)]^2\,g'(x) + f(x)g(x)f'(x) + f(x)g(x)g'(x) + [g(x)]^2\,f'(x) - f(x)g(x)f'(x) - f(x)g(x)g'(x)}{[f(x) + g(x)]^2}$$

$$= \frac{f'(x)[g(x)]^2 + g'(x)[f(x)]^2}{[f(x) + g(x)]^2}$$

58. Using the Chain Rule repeatedly, $h(x) = f(g(\sin 4x)) \quad \Rightarrow$

$$h'(x) = f'(g(\sin 4x)) \cdot \frac{d}{dx}(g(\sin 4x)) = f'(g(\sin 4x)) \cdot g'(\sin 4x) \cdot \frac{d}{dx}(\sin 4x) = f'(g(\sin 4x))g'(\sin 4x)(\cos 4x)(4).$$

59. $y = [\ln(x+4)]^2 \quad \Rightarrow \quad y' = 2[\ln(x+4)]^1 \cdot \dfrac{1}{x+4} \cdot 1 = 2\,\dfrac{\ln(x+4)}{x+4}$ and $y' = 0 \quad \Leftrightarrow \quad \ln(x+4) = 0 \quad \Leftrightarrow$

$x + 4 = e^0 \quad \Rightarrow \quad x + 4 = 1 \quad \Leftrightarrow \quad x = -3$, so the tangent is horizontal at the point $(-3, 0)$.

60. (a) The line $x - 4y = 1$ has slope $\frac{1}{4}$. A tangent to $y = e^x$ has slope $\frac{1}{4}$ when $y' = e^x = \frac{1}{4} \quad \Rightarrow \quad x = \ln\frac{1}{4} = -\ln 4$. Since $y = e^x$, the y-coordinate is $\frac{1}{4}$ and the point of tangency is $\left(-\ln 4, \frac{1}{4}\right)$. Thus, an equation of the tangent line is $y - \frac{1}{4} = \frac{1}{4}(x + \ln 4)$ or $y = \frac{1}{4}x + \frac{1}{4}(\ln 4 + 1)$.

(b) The slope of the tangent at the point (a, e^a) is $\left.\dfrac{d}{dx}e^x\right|_{x=a} = e^a$. Thus, an equation of the tangent line is $y - e^a = e^a(x - a)$. We substitute $x = 0, y = 0$ into this equation, since we want the line to pass through the origin: $0 - e^a = e^a(0 - a) \quad \Leftrightarrow \quad -e^a = e^a(-a) \quad \Leftrightarrow \quad a = 1$. So an equation of the tangent line at the point $(a, e^a) = (1, e)$ is $y - e = e(x - 1)$ or $y = ex$.

61. $x^2 + 2y^2 = 1 \quad \Rightarrow \quad 2x + 4yy' = 0 \quad \Rightarrow \quad y' = -x/(2y) = 1 \quad \Leftrightarrow \quad x = -2y$. Since the points lie on the ellipse, we have $(-2y)^2 + 2y^2 = 1 \quad \Rightarrow \quad 6y^2 = 1 \quad \Rightarrow \quad y = \pm\frac{1}{\sqrt{6}}$. The points are $\left(-\frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right)$ and $\left(\frac{2}{\sqrt{6}}, -\frac{1}{\sqrt{6}}\right)$.

62. (a) $f(x) = \dfrac{\ln x}{x} \quad \Rightarrow \quad f'(x) = \dfrac{x(1/x) - (\ln x)\cdot 1}{x^2} = \dfrac{1 - \ln x}{x^2}$. $f'(x) > 0 \quad \Rightarrow \quad 1 - \ln x > 0 \quad \Rightarrow$ $\ln x < 1 \quad \Rightarrow \quad x < e$. Since the domain of f is $(0, \infty)$, f is increasing on $(0, e)$.

(b) $f''(x) = \dfrac{x^2(-1/x) - (1 - \ln x)(2x)}{(x^2)^2} = \dfrac{x[-1 - 2(1 - \ln x)]}{x^4} = \dfrac{2\ln x - 3}{x^3}$.

$f''(x) > 0 \quad \Rightarrow \quad 2\ln x - 3 > 0 \quad \Rightarrow \quad \ln x > \frac{3}{2} \quad \Rightarrow \quad x > e^{3/2} \approx 4.48$. f is concave upward on $\left(e^{3/2}, \infty\right)$.

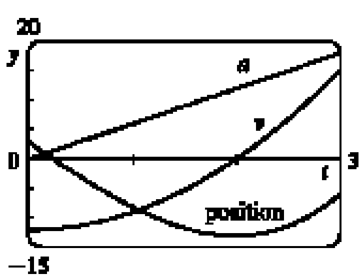
63. $y = f(x) = ax^2 + bx + c \quad \Rightarrow \quad f'(x) = 2ax + b$. We know that $f'(-1) = 6$ and $f'(5) = -2$, so $-2a + b = 6$ and $10a + b = -2$. Subtracting the first equation from the second gives $12a = -8 \quad \Rightarrow \quad a = -\frac{2}{3}$. Substituting $-\frac{2}{3}$ for a in the first equation gives $b = \frac{14}{3}$. Now $f(1) = 4 \quad \Rightarrow \quad 4 = a + b + c$, so $c = 4 + \frac{2}{3} - \frac{14}{3} = 0$ and hence, $f(x) = -\frac{2}{3}x^2 + \frac{14}{3}x$.

64. (a) $y = t^3 - 12t + 3 \quad \Rightarrow \quad v(t) = y' = 3t^2 - 12 \quad \Rightarrow \quad a(t) = v'(t) = 6t$

(b) $v(t) = 3(t^2 - 4) > 0$ when $t > 2$, so it moves upward when $t > 2$ and downward when $0 \le t < 2$.

(c) Distance upward $= y(3) - y(2) = -6 - (-13) = 7$,
Distance downward $= y(0) - y(2) = 3 - (-13) = 16$. Total distance $= 7 + 16 = 23$.

(d)



(e) The particle is speeding up when v and a have the same sign, that is, when $t > 2$. The particle is slowing down when v and a have opposite signs; that is, when $0 < t < 2$.

65. $s(t) = Ae^{-ct}\cos(\omega t + \delta) \quad \Rightarrow$

$$\begin{aligned} v(t) = s'(t) &= A\{e^{-ct}[-\omega\sin(\omega t+\delta)] + \cos(\omega t+\delta)(-ce^{-ct})\} \\ &= -Ae^{-ct}[\omega\sin(\omega t+\delta) + c\cos(\omega t+\delta)] \quad \Rightarrow \end{aligned}$$

$$\begin{aligned} a(t) = v'(t) &= -A\{e^{-ct}[\omega^2\cos(\omega t+\delta) - c\omega\sin(\omega t+\delta)] + [\omega\sin(\omega t+\delta) + c\cos(\omega t+\delta)](-ce^{-ct})\} \\ &= -Ae^{-ct}[\omega^2\cos(\omega t+\delta) - c\omega\sin(\omega t+\delta) - c\omega\sin(\omega t+\delta) - c^2\cos(\omega t+\delta)] \\ &= -Ae^{-ct}[(\omega^2 - c^2)\cos(\omega t+\delta) - 2c\omega\sin(\omega t+\delta)] \\ &= Ae^{-ct}[(c^2 - \omega^2)\cos(\omega t+\delta) + 2c\omega\sin(\omega t+\delta)] \end{aligned}$$

66. (a) $x = \sqrt{b^2 + c^2t^2} \quad \Rightarrow \quad v(t) = x' = \left[1/\left(2\sqrt{b^2+c^2t^2}\right)\right]2c^2t = c^2t/\sqrt{b^2+c^2t^2} \quad \Rightarrow$

$$a(t) = v'(t) = \frac{c^2\sqrt{b^2+c^2t^2} - c^2t\left(c^2t/\sqrt{b^2+c^2t^2}\right)}{b^2+c^2t^2} = \frac{b^2c^2}{(b^2+c^2t^2)^{3/2}}$$

(b) $v(t) > 0$ for $t > 0$, so the particle always moves in the positive direction.

67. The linear density ρ is the rate of change of mass m with respect to length x. $m = x(1+\sqrt{x}) = x + x^{3/2} \quad \Rightarrow$
$\rho = dm/dx = 1 + \frac{3}{2}\sqrt{x}$, so the linear density when $x = 4$ is $1 + \frac{3}{2}\sqrt{4} = 4$ kg/m.

68. (a) $V = \frac{1}{3}\pi r^2 h \quad \Rightarrow \quad dV/dh = \frac{1}{3}\pi r^2 \quad$ [r constant]

(b) $V = \frac{1}{3}\pi r^2 h \quad \Rightarrow \quad dV/dr = \frac{2}{3}\pi rh \quad$ [h constant]

69. (a) $C(x) = 920 + 2x - 0.02x^2 + 0.00007x^3 \quad \Rightarrow \quad C'(x) = 2 - 0.04x + 0.00021x^2$

(b) $C'(100) = 2 - 4 + 2.1 = \$0.10/\text{unit}$. This value represents the rate at which costs are increasing as the hundredth unit is produced, and is the approximate cost of producing the 101st unit.

(c) The cost of producing the 101st item is $C(101) - C(100) = 990.10107 - 990 = \$0.10107$, slightly larger than $C'(100)$.

(d) $C''(x) = -0.04 + 0.00042x = 0 \quad \Rightarrow \quad x = \frac{0.04}{0.00042} \approx 95.24$ and C'' changes from negative to positive at this value of x. This is the value of x at which the marginal cost is minimized.

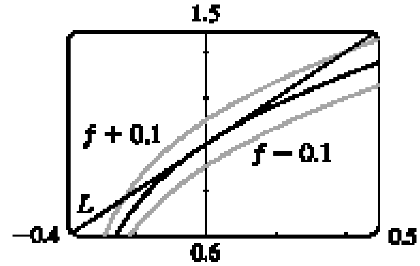
70. (a) $\lim\limits_{t\to\infty} C(t) = \lim\limits_{t\to\infty}[K(e^{-at} - e^{-bt})] = K\lim\limits_{t\to\infty}(e^{-at} - e^{-bt}) = K(0-0) = 0$ because $-at \to -\infty$ and $-bt \to -\infty$ as $t \to \infty$.

(b) $C(t) = K(e^{-at} - e^{-bt}) \quad \Rightarrow \quad C'(t) = K(e^{-at}(-a) - e^{-bt}(-b)) = K(-ae^{-at} + be^{-bt})$

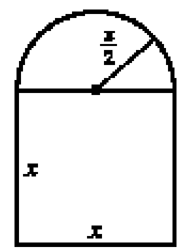
(c) $C'(t) = 0 \quad \Rightarrow \quad be^{-bt} = ae^{-at} \quad \Rightarrow \quad \dfrac{b}{a} = e^{(-a+b)t} \quad \Rightarrow \quad \ln\dfrac{b}{a} = (b-a)t \quad \Rightarrow \quad t = \dfrac{\ln(b/a)}{b-a}$

71. (a) $f(x) = \sqrt[3]{1+3x} = (1+3x)^{1/3} \Rightarrow f'(x) = (1+3x)^{-2/3}$, so the linearization of f at $a=0$ is $L(x) = f(0) + f'(0)(x-0) = 1^{1/3} + 1^{-2/3}x = 1 + x$. Thus, $\sqrt[3]{1+3x} \approx 1 + x \Rightarrow$ $\sqrt[3]{1.03} = \sqrt[3]{1+3(0.01)} \approx 1 + (0.01) = 1.01$.

(b) The linear approximation is $\sqrt[3]{1+3x} \approx 1 + x$, so for the required accuracy we want $\sqrt[3]{1+3x} - 0.1 < 1 + x < \sqrt[3]{1+3x} + 0.1$. From the graph, it appears that this is true when $-0.23 < x < 0.40$.



72. $A = x^2 + \frac{1}{2}\pi\left(\frac{1}{2}x\right)^2 = \left(1 + \frac{\pi}{8}\right)x^2 \Rightarrow dA = \left(2 + \frac{\pi}{4}\right)x\,dx$. When $x = 60$ and $dx = 0.1$, $dA = \left(2 + \frac{\pi}{4}\right)60(0.1) = 12 + \frac{3\pi}{2}$, so the maximum error is approximately $12 + \frac{3\pi}{2} \approx 16.7\text{ cm}^2$.



73. $\displaystyle\lim_{\theta\to\pi/3} \frac{\cos\theta - 0.5}{\theta - \pi/3} = \left[\frac{d}{d\theta}\cos\theta\right]_{\theta=\pi/3} = -\sin\frac{\pi}{3} = -\frac{\sqrt{3}}{2}$

74. $\frac{d}{dx}[f(2x)] = x^2 \Rightarrow f'(2x)\cdot 2 = x^2 \Rightarrow f'(2x) = \frac{1}{2}x^2$. Let $t = 2x$. Then $f'(t) = \frac{1}{2}\left(\frac{1}{2}t\right)^2 = \frac{1}{8}t^2$, so $f'(x) = \frac{1}{8}x^2$.

75.
$$\lim_{x\to 0}\frac{\sqrt{1+\tan x} - \sqrt{1+\sin x}}{x^3} = \lim_{x\to 0}\frac{\left(\sqrt{1+\tan x} - \sqrt{1+\sin x}\right)\left(\sqrt{1+\tan x} + \sqrt{1+\sin x}\right)}{x^3\left(\sqrt{1+\tan x} + \sqrt{1+\sin x}\right)}$$
$$= \lim_{x\to 0}\frac{(1+\tan x) - (1+\sin x)}{x^3\left(\sqrt{1+\tan x} + \sqrt{1+\sin x}\right)} = \lim_{x\to 0}\frac{\sin x\,(1/\cos x - 1)}{x^3\left(\sqrt{1+\tan x} + \sqrt{1+\sin x}\right)}\cdot\frac{\cos x}{\cos x}$$
$$= \lim_{x\to 0}\frac{\sin x\,(1 - \cos x)}{x^3\left(\sqrt{1+\tan x} + \sqrt{1+\sin x}\right)\cos x}\cdot\frac{1+\cos x}{1+\cos x}$$
$$= \lim_{x\to 0}\frac{\sin x\cdot\sin^2 x}{x^3\left(\sqrt{1+\tan x} + \sqrt{1+\sin x}\right)\cos x\,(1+\cos x)}$$
$$= \left(\lim_{x\to 0}\frac{\sin x}{x}\right)^3 \lim_{x\to 0}\frac{1}{\left(\sqrt{1+\tan x} + \sqrt{1+\sin x}\right)\cos x\,(1+\cos x)}$$
$$= 1^3\cdot\frac{1}{\left(\sqrt{1}+\sqrt{1}\right)\cdot 1\cdot(1+1)} = \frac{1}{4}$$

76. Let (b, c) be on the curve, that is, $b^{2/3} + c^{2/3} = a^{2/3}$. Now $x^{2/3} + y^{2/3} = a^{2/3} \Rightarrow \frac{2}{3}x^{-1/3} + \frac{2}{3}y^{-1/3}\frac{dy}{dx} = 0$, so $\frac{dy}{dx} = -\frac{y^{1/3}}{x^{1/3}} = -\left(\frac{y}{x}\right)^{1/3}$, so at (b, c) the slope of the tangent line is $-(c/b)^{1/3}$ and an equation of the tangent line is $y - c = -(c/b)^{1/3}(x - b)$ or $y = -(c/b)^{1/3}x + (c + b^{2/3}c^{1/3})$. Setting $y = 0$, we find that the x-intercept is $b^{1/3}c^{2/3} + b = b^{1/3}(c^{2/3} + b^{2/3})$ and setting $x = 0$ we find that the y-intercept is $c + b^{2/3}c^{1/3} = c^{1/3}(c^{2/3} + b^{2/3})$. So the length of the tangent line between these two points is

$$\sqrt{[b^{1/3}(c^{2/3} + b^{2/3})]^2 + [c^{1/3}(c^{2/3} + b^{2/3})]^2} = \sqrt{b^{2/3}(a^{2/3})^2 + c^{2/3}(a^{2/3})^2} = \sqrt{(b^{2/3} + c^{2/3})a^{4/3}}$$

$$= \sqrt{a^{2/3}a^{4/3}} = \sqrt{a^2} = a = \text{constant}$$

1. (a) The Power Rule: If n is any real number, then $\frac{d}{dx}(x^n) = nx^{n-1}$. The derivative of a variable base raised to a constant power is the power times the base raised to the power minus one.

(b) The Constant Multiple Rule: If c is a constant and f is a differentiable function, then $\frac{d}{dx}[cf(x)] = c\frac{d}{dx}f(x)$. The derivative of a constant times a function is the constant times the derivative of the function.

(c) The Sum Rule: If f and g are both differentiable, then $\frac{d}{dx}[f(x) + g(x)] = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$. The derivative of a sum of functions is the sum of the derivatives.

(d) The Difference Rule: If f and g are both differentiable, then $\frac{d}{dx}[f(x) - g(x)] = \frac{d}{dx}f(x) - \frac{d}{dx}g(x)$. The derivative of a difference of functions is the difference of the derivatives.

(e) The Product Rule: If f and g are both differentiable, then $\frac{d}{dx}[f(x)g(x)] = f(x)\frac{d}{dx}g(x) + g(x)\frac{d}{dx}f(x)$. The derivative of a product of two functions is the first function times the derivative of the second function plus the second function times the derivative of the first function.

(f) The Quotient Rule: If f and g are both differentiable, then $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\frac{d}{dx}f(x) - f(x)\frac{d}{dx}g(x)}{[g(x)]^2}$.
The derivative of a quotient of functions is the denominator times the derivative of the numerator minus the numerator times the derivative of the denominator, all divided by the square of the denominator.

(g) The Chain Rule: If f and g are both differentiable and $F = f \circ g$ is the composite function defined by $F(x) = f(g(x))$, then F is differentiable and F' is given by the product $F'(x) = f'(g(x))g'(x)$. The derivative of a composite function is the derivative of the outer function evaluated at the inner function times the derivative of the inner function.

Name: ___________________ Class:___ Date: _____________

2. (a) $y = x^n \Rightarrow y' = nx^{n-1}$

(b) $y = e^x \Rightarrow y' = e^x$

(c) $y = a^x \Rightarrow y' = a^x \ln a$

(d) $y = \ln x \Rightarrow y' = 1/x$

(e) $y = \log_a x \Rightarrow y' = 1/(x \ln a)$

(f) $y = \sin x \Rightarrow y' = \cos x$

(g) $y = \cos x \Rightarrow y' = -\sin x$

(h) $y = \tan x \Rightarrow y' = \sec^2 x$

(i) $y = \csc x \Rightarrow y' = -\csc x \cot x$

(j) $y = \sec x \Rightarrow y' = \sec x \tan x$

(k) $y = \cot x \Rightarrow y' = -\csc^2 x$

(l) $y = \sin^{-1} x \Rightarrow y' = 1/\sqrt{1 - x^2}$

(m) $y = \tan^{-1} x \Rightarrow y' = 1/(1 + x^2)$

3. (a) e is the number such that $\lim\limits_{h \to 0} \dfrac{e^h - 1}{h} = 1$.

(b) $e = \lim\limits_{x \to 0} (1 + x)^{1/x}$

(c) The differentiation formula for $y = a^x$ $[y' = a^x \ln a]$ is simplest when $a = e$ because $\ln e = 1$.

(d) The differentiation formula for $y = \log_a x$ $[y' = 1/(x \ln a)]$ is simplest when $a = e$ because $\ln e = 1$.

4. (a) Implicit differentiation consists of differentiating both sides of an equation involving x and y with respect to x, and then solving the resulting equation for y'.

(b) Logarithmic differentiation consists of taking natural logarithms of both sides of an equation $y = f(x)$, simplifying, differentiating implicitly with respect to x, and then solving the resulting equation for y'.

5. The linearization L of f at $x = a$ is $L(x) = f(a) + f'(a)(x - a)$.

1. True. This is the Sum Rule.

2. False. See the warning before the Product Rule.

3. True. This is the Chain Rule.

4. True by the Chain Rule.

5. False. $\dfrac{d}{dx} f(\sqrt{x}\,) = \dfrac{f'(\sqrt{x}\,)}{2\sqrt{x}}$ by the Chain Rule.

6. False. e^2 is a constant, so $y' = 0$.

7. False. $\frac{d}{dx}10^x = 10^x \ln 10$

8. False. $\ln 10$ is a constant, so its derivative is 0.

9. True. $\frac{d}{dx}(\tan^2 x) = 2\tan x \sec^2 x$, and $\frac{d}{dx}(\sec^2 x) = 2\sec x(\sec x \tan x) = 2\tan x \sec^2 x$.

10. False. $f(x) = |x^2 + x| = x^2 + x$ for $x \geq 0$ or $x \leq -1$ and $|x^2 + x| = -(x^2 + x)$ for $-1 < x < 0$. So $f'(x) = 2x + 1$ for $x > 0$ or $x < -1$ and $f'(x) = -(2x + 1)$ for $-1 < x < 0$. But $|2x + 1| = 2x + 1$ for $x \geq -\frac{1}{2}$ and $|2x + 1| = -2x - 1$ for $x < -\frac{1}{2}$.

11. True. $g(x) = x^5 \Rightarrow g'(x) = 5x^4 \Rightarrow g'(2) = 5(2)^4 = 80$, and by the definition of the derivative, $\lim_{x \to 2} \frac{g(x) - g(2)}{x - 2} = g'(2) = 80$.

12. False. A tangent line to the parabola $y = x^2$ has slope $dy/dx = 2x$, so at $(-2, 4)$ the slope of the tangent is $2(-2) = -4$ and an equation of the tangent line is $y - 4 = -4(x + 2)$. [The given equation, $y - 4 = 2x(x + 2)$, is not even linear!]