

1. (a) (i) $\lim_{x \rightarrow 2^+} f(x) = 3$ (ii) $\lim_{x \rightarrow -3^+} f(x) = 0$
 (iii) $\lim_{x \rightarrow -3} f(x)$ does not exist since the left and right limits are not equal. (The left limit is -2 .)
 (iv) $\lim_{x \rightarrow 4} f(x) = 2$
 (v) $\lim_{x \rightarrow 0} f(x) = \infty$ (vi) $\lim_{x \rightarrow 2^-} f(x) = -\infty$
 (vii) $\lim_{x \rightarrow \infty} f(x) = 4$ (viii) $\lim_{x \rightarrow -\infty} f(x) = -1$

(b) The equations of the horizontal asymptotes are $y = -1$ and $y = 4$.

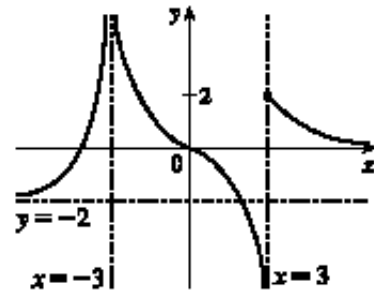
(c) The equations of the vertical asymptotes are $x = 0$ and $x = 2$.

(d) f is discontinuous at $x = -3, 0, 2,$ and 4 . The discontinuities are jump, infinite, infinite, and removable, respectively.

2. $\lim_{x \rightarrow -\infty} f(x) = -2, \quad \lim_{x \rightarrow \infty} f(x) = 0, \quad \lim_{x \rightarrow -3} f(x) = \infty,$

$\lim_{x \rightarrow 3^-} f(x) = -\infty, \quad \lim_{x \rightarrow 3^+} f(x) = 2,$

f is continuous from the right at 3



3. Since the exponential function is continuous, $\lim_{x \rightarrow 1} e^{x^3 - x} = e^{1-1} = e^0 = 1$.

4. Since rational functions are continuous, $\lim_{x \rightarrow 3} \frac{x^2 - 9}{x^2 + 2x - 3} = \frac{3^2 - 9}{3^2 + 2(3) - 3} = \frac{0}{12} = 0$.

5. $\lim_{x \rightarrow -3} \frac{x^2 - 9}{x^2 + 2x - 3} = \lim_{x \rightarrow -3} \frac{(x+3)(x-3)}{(x+3)(x-1)} = \lim_{x \rightarrow -3} \frac{x-3}{x-1} = \frac{-3-3}{-3-1} = \frac{-6}{-4} = \frac{3}{2}$

6. $\lim_{x \rightarrow 1^+} \frac{x^2 - 9}{x^2 + 2x - 3} = -\infty$ since $x^2 + 2x - 3 \rightarrow 0$ as $x \rightarrow 1^+$ and $\frac{x^2 - 9}{x^2 + 2x - 3} < 0$ for $1 < x < 3$.

7. $\lim_{h \rightarrow 0} \frac{(h-1)^3 + 1}{h} = \lim_{h \rightarrow 0} \frac{(h^3 - 3h^2 + 3h - 1) + 1}{h} = \lim_{h \rightarrow 0} \frac{h^3 - 3h^2 + 3h}{h} = \lim_{h \rightarrow 0} (h^2 - 3h + 3) = 3$

Another solution: Factor the numerator as a sum of two cubes and then simplify.

$$\begin{aligned} \lim_{h \rightarrow 0} \frac{(h-1)^3 + 1}{h} &= \lim_{h \rightarrow 0} \frac{(h-1)^3 + 1^3}{h} = \lim_{h \rightarrow 0} \frac{[(h-1) + 1][(h-1)^2 - 1(h-1) + 1^2]}{h} \\ &= \lim_{h \rightarrow 0} [(h-1)^2 - h + 2] = 1 - 0 + 2 = 3 \end{aligned}$$

$$8. \lim_{t \rightarrow 2} \frac{t^2 - 4}{t^3 - 8} = \lim_{t \rightarrow 2} \frac{(t+2)(t-2)}{(t-2)(t^2 + 2t + 4)} = \lim_{t \rightarrow 2} \frac{t+2}{t^2 + 2t + 4} = \frac{2+2}{4+4+4} = \frac{4}{12} = \frac{1}{3}$$

$$9. \lim_{r \rightarrow 9} \frac{\sqrt{r}}{(r-9)^4} = \infty \text{ since } (r-9)^4 \rightarrow 0 \text{ as } r \rightarrow 9 \text{ and } \frac{\sqrt{r}}{(r-9)^4} > 0 \text{ for } r \neq 9.$$

$$10. \lim_{v \rightarrow 4^+} \frac{4-v}{|4-v|} = \lim_{v \rightarrow 4^+} \frac{4-v}{-(4-v)} = \lim_{v \rightarrow 4^+} \frac{1}{-1} = -1$$

$$11. \text{ Let } t = \sin x. \text{ Then as } x \rightarrow \pi^-, \sin x \rightarrow 0^+, \text{ so } t \rightarrow 0^+. \text{ Thus, } \lim_{x \rightarrow \pi^-} \ln(\sin x) = \lim_{t \rightarrow 0^+} \ln t = -\infty.$$

$$12. \lim_{x \rightarrow -\infty} \frac{1 - 2x^2 - x^4}{5 + x - 3x^4} = \lim_{x \rightarrow -\infty} \frac{(1 - 2x^2 - x^4)/x^4}{(5 + x - 3x^4)/x^4} = \lim_{x \rightarrow -\infty} \frac{1/x^4 - 2/x^2 - 1}{5/x^4 + 1/x^3 - 3} = \frac{0 - 0 - 1}{0 + 0 - 3} = \frac{-1}{-3} = \frac{1}{3}$$

$$\begin{aligned} 13. \lim_{x \rightarrow \infty} (\sqrt{x^2 + 4x + 1} - x) &= \lim_{x \rightarrow \infty} \left[\frac{\sqrt{x^2 + 4x + 1} - x}{1} \cdot \frac{\sqrt{x^2 + 4x + 1} + x}{\sqrt{x^2 + 4x + 1} + x} \right] \\ &= \lim_{x \rightarrow \infty} \frac{(x^2 + 4x + 1) - x^2}{\sqrt{x^2 + 4x + 1} + x} \\ &= \lim_{x \rightarrow \infty} \frac{(4x + 1)/x}{(\sqrt{x^2 + 4x + 1} + x)/x} \quad \left[\text{divide by } x = \sqrt{x^2} \text{ for } x > 0 \right] \\ &= \lim_{x \rightarrow \infty} \frac{4 + 1/x}{\sqrt{1 + 4/x + 1/x^2} + 1} = \frac{4 + 0}{\sqrt{1 + 0 + 0} + 1} = \frac{4}{2} = 2 \end{aligned}$$

$$14. \text{ Let } t = x - x^2 = x(1 - x). \text{ Then as } x \rightarrow \infty, t \rightarrow -\infty, \text{ and } \lim_{x \rightarrow \infty} e^{x-x^2} = \lim_{t \rightarrow -\infty} e^t = 0.$$

$$15. \text{ Let } t = 1/x. \text{ Then as } x \rightarrow 0^+, t \rightarrow \infty, \text{ and } \lim_{x \rightarrow 0^+} \tan^{-1}(1/x) = \lim_{t \rightarrow \infty} \tan^{-1} t = \frac{\pi}{2}.$$

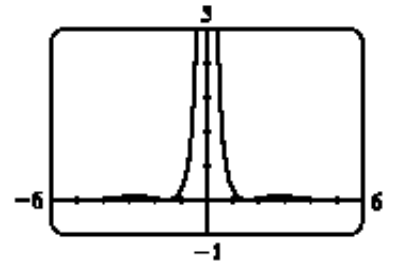
$$\begin{aligned} 16. \lim_{x \rightarrow 1} \left(\frac{1}{x-1} + \frac{1}{x^2 - 3x + 2} \right) &= \lim_{x \rightarrow 1} \left[\frac{1}{x-1} + \frac{1}{(x-1)(x-2)} \right] = \lim_{x \rightarrow 1} \left[\frac{x-2}{(x-1)(x-2)} + \frac{1}{(x-1)(x-2)} \right] \\ &= \lim_{x \rightarrow 1} \left[\frac{x-1}{(x-1)(x-2)} \right] = \lim_{x \rightarrow 1} \frac{1}{x-2} = \frac{1}{1-2} = -1 \end{aligned}$$

17. From the graph of $y = (\cos^2 x)/x^2$, it appears that $y = 0$ is the horizontal asymptote and $x = 0$ is the vertical asymptote. Now $0 \leq (\cos x)^2 \leq 1 \Rightarrow$

$$\frac{0}{x^2} \leq \frac{\cos^2 x}{x^2} \leq \frac{1}{x^2} \Rightarrow 0 \leq \frac{\cos^2 x}{x^2} \leq \frac{1}{x^2}. \text{ But } \lim_{x \rightarrow \pm\infty} 0 = 0 \text{ and}$$

$$\lim_{x \rightarrow \pm\infty} \frac{1}{x^2} = 0, \text{ so by the Squeeze Theorem, } \lim_{x \rightarrow \pm\infty} \frac{\cos^2 x}{x^2} = 0.$$

Thus, $y = 0$ is the horizontal asymptote. $\lim_{x \rightarrow 0} \frac{\cos^2 x}{x^2} = \infty$ because $\cos^2 x \rightarrow 1$ and $x^2 \rightarrow 0$ as $x \rightarrow 0$, so $x = 0$ is the vertical asymptote.



18. From the graph of $y = f(x) = \sqrt{x^2 + x + 1} - \sqrt{x^2 - x}$, it appears that there are 2 horizontal asymptotes and possibly 2 vertical asymptotes. To obtain a different form for f , let's multiply and divide it by its conjugate.

$$\begin{aligned} f_1(x) &= \left(\sqrt{x^2 + x + 1} - \sqrt{x^2 - x} \right) \frac{\sqrt{x^2 + x + 1} + \sqrt{x^2 - x}}{\sqrt{x^2 + x + 1} + \sqrt{x^2 - x}} \\ &= \frac{(x^2 + x + 1) - (x^2 - x)}{\sqrt{x^2 + x + 1} + \sqrt{x^2 - x}} = \frac{2x + 1}{\sqrt{x^2 + x + 1} + \sqrt{x^2 - x}} \end{aligned}$$

Now

$$\begin{aligned} \lim_{x \rightarrow \infty} f_1(x) &= \lim_{x \rightarrow \infty} \frac{2x + 1}{\sqrt{x^2 + x + 1} + \sqrt{x^2 - x}} \\ &= \lim_{x \rightarrow \infty} \frac{2 + (1/x)}{\sqrt{1 + (1/x) + (1/x^2)} + \sqrt{1 - (1/x)}} \quad [\text{since } \sqrt{x^2} = x \text{ for } x > 0] \\ &= \frac{2}{1 + 1} = 1, \end{aligned}$$

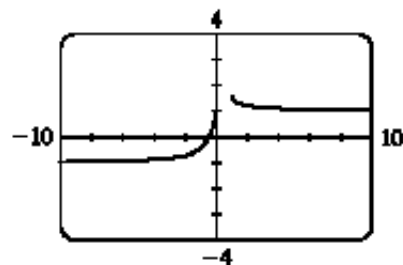
so $y = 1$ is a horizontal asymptote. For $x < 0$, we have $\sqrt{x^2} = |x| = -x$, so when we divide the denominator by x , with $x < 0$, we get

$$\frac{\sqrt{x^2 + x + 1} + \sqrt{x^2 - x}}{x} = -\frac{\sqrt{x^2 + x + 1} + \sqrt{x^2 - x}}{\sqrt{x^2}} = -\left[\sqrt{1 + \frac{1}{x} + \frac{1}{x^2}} + \sqrt{1 - \frac{1}{x}} \right]$$

Therefore,

$$\begin{aligned} \lim_{x \rightarrow -\infty} f_1(x) &= \lim_{x \rightarrow -\infty} \frac{2x + 1}{\sqrt{x^2 + x + 1} + \sqrt{x^2 - x}} = \lim_{x \rightarrow -\infty} \frac{2 + (1/x)}{-\left[\sqrt{1 + (1/x) + (1/x^2)} + \sqrt{1 - (1/x)} \right]} \\ &= \frac{2}{-(1 + 1)} = -1, \end{aligned}$$

so $y = -1$ is a horizontal asymptote. As $x \rightarrow 0^-$, $f(x) \rightarrow 1$, so $x = 0$ is *not* a vertical asymptote. As $x \rightarrow 1^+$, $f(x) \rightarrow \sqrt{3}$, so $x = 1$ is *not* a vertical asymptote and hence there are no vertical asymptotes.



19. Since $2x - 1 \leq f(x) \leq x^2$ for $0 < x < 3$ and $\lim_{x \rightarrow 1} (2x - 1) = 1 = \lim_{x \rightarrow 1} x^2$, we have $\lim_{x \rightarrow 1} f(x) = 1$ by the Squeeze Theorem.

20. Let $f(x) = -x^2$, $g(x) = x^2 \cos(1/x^2)$ and $h(x) = x^2$. Then since $|\cos(1/x^2)| \leq 1$ for $x \neq 0$, we have $f(x) \leq g(x) \leq h(x)$ for $x \neq 0$, and so $\lim_{x \rightarrow 0} f(x) = \lim_{x \rightarrow 0} h(x) = 0 \Rightarrow \lim_{x \rightarrow 0} g(x) = 0$ by the Squeeze Theorem.

21. (a) $f(x) = \sqrt{-x}$ if $x < 0$, $f(x) = 3 - x$ if $0 \leq x < 3$, $f(x) = (x - 3)^2$ if $x > 3$.

(i) $\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} (3 - x) = 3$

(ii) $\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} \sqrt{-x} = 0$

(iii) Because of (i) and (ii), $\lim_{x \rightarrow 0} f(x)$ does not exist.

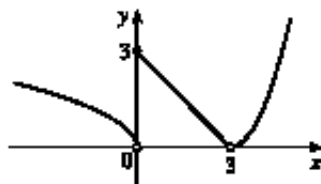
(iv) $\lim_{x \rightarrow 3^-} f(x) = \lim_{x \rightarrow 3^-} (3 - x) = 0$

(v) $\lim_{x \rightarrow 3^+} f(x) = \lim_{x \rightarrow 3^+} (x - 3)^2 = 0$

(vi) Because of (iv) and (v), $\lim_{x \rightarrow 3} f(x) = 0$.

(b) f is discontinuous at 0 since $\lim_{x \rightarrow 0} f(x)$ does not exist.

(c)



f is discontinuous at 3 since $f(3)$ does not exist.

22. (a) $x^2 - 9$ is continuous on \mathbb{R} since it is a polynomial and \sqrt{x} is continuous on $[0, \infty)$, so the composition $\sqrt{x^2 - 9}$ is continuous on $\{x \mid x^2 - 9 \geq 0\} = (-\infty, -3] \cup [3, \infty)$. Note that $x^2 - 2 \neq 0$ on this set and so the quotient function

$$g(x) = \frac{\sqrt{x^2 - 9}}{x^2 - 2}$$

is continuous on its domain, $(-\infty, -3] \cup [3, \infty)$.

(b) f is discontinuous at 0 since $\lim_{x \rightarrow 0} f(x)$ does not exist. f is discontinuous at 3 since $f(3)$ does not exist.

23. $f(x) = 2x^3 + x^2 + 2$ is a polynomial, so it is continuous on $[-2, -1]$ and $f(-2) = -10 < 0 < 1 = f(-1)$. So by the Intermediate Value Theorem there is a number c in $(-2, -1)$ such that $f(c) = 0$, that is, the equation $2x^3 + x^2 + 2 = 0$ has a root in $(-2, -1)$.

24. $f(x) = e^{-x^2} - x$ is continuous on \mathbb{R} so it is continuous on $[0, 1]$. $f(0) = 1 > 0 > 1/e - 1 = f(1)$. So by the Intermediate Value Theorem, there is a number c in $(0, 1)$ such that $f(c) = 0$. Thus, $e^{-x^2} - x = 0$, or $e^{-x^2} = x$, has a root in $(0, 1)$.

25. (a) $s = s(t) = 1 + 2t + t^2/4$. The average velocity over the time interval $[1, 1 + h]$ is

$$v_{\text{ave}} = \frac{s(1+h) - s(1)}{(1+h) - 1} = \frac{1 + 2(1+h) + (1+h)^2/4 - 13/4}{h} = \frac{10h + h^2}{4h} = \frac{10 + h}{4}.$$

So for the following intervals the average velocities are:

(i) $[1, 3]$: $h = 2$, $v_{\text{ave}} = (10 + 2)/4 = 3$ m/s

(ii) $[1, 2]$: $h = 1$, $v_{\text{ave}} = (10 + 1)/4 = 2.75$ m/s

(iii) $[1, 1.5]$: $h = 0.5$, $v_{\text{ave}} = (10 + 0.5)/4 = 2.625$ m/s

(iv) $[1, 1.1]$: $h = 0.1$, $v_{\text{ave}} = (10 + 0.1)/4 = 2.525$ m/s

(b) When $t = 1$, the instantaneous velocity is $\lim_{h \rightarrow 0} \frac{s(1+h) - s(1)}{h} = \lim_{h \rightarrow 0} \frac{10 + h}{4} = \frac{10}{4} = 2.5$ m/s.

26. (a) When V increases from 200 in^3 to 250 in^3 , we have $\Delta V = 250 - 200 = 50 \text{ in}^3$, and since $P = 800/V$,

$$\Delta P = P(250) - P(200) = \frac{800}{250} - \frac{800}{200} = 3.2 - 4 = -0.8 \text{ lb/in}^2. \text{ So the average rate of change}$$

$$\text{is } \frac{\Delta P}{\Delta V} = \frac{-0.8}{50} = -0.016 \frac{\text{lb/in}^2}{\text{in}^3}.$$

- (b) Since $V = 800/P$, the instantaneous rate of change of V with respect to P is

$$\begin{aligned} \lim_{h \rightarrow 0} \frac{\Delta V}{\Delta P} &= \lim_{h \rightarrow 0} \frac{V(P+h) - V(P)}{h} = \lim_{h \rightarrow 0} \frac{800/(P+h) - 800/P}{h} \\ &= \lim_{h \rightarrow 0} \frac{800[P - (P+h)]}{h(P+h)P} = \lim_{h \rightarrow 0} \frac{-800}{(P+h)P} = -\frac{800}{P^2} \end{aligned}$$

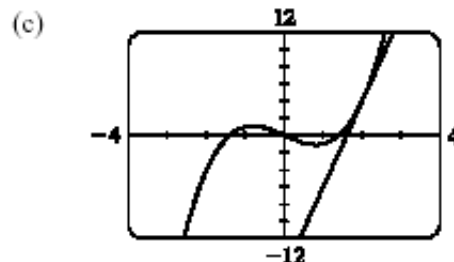
which is inversely proportional to the square of P .

27. Estimating the slopes of the tangent lines at $x = 2, 3,$ and 5 , we obtain approximate values $0.4, 2,$ and 0.1 . Since the graph is concave downward at $x = 5$, $f''(5)$ is negative. Arranging the numbers in increasing order, we have:

$$f''(5) < 0 < f'(5) < f'(2) < 1 < f'(3).$$

$$\begin{aligned} 28. \text{ (a) } f'(2) &= \lim_{x \rightarrow 2} \frac{f(x) - f(2)}{x - 2} = \lim_{x \rightarrow 2} \frac{x^3 - 2x - 4}{x - 2} \\ &= \lim_{x \rightarrow 2} \frac{(x - 2)(x^2 + 2x + 2)}{x - 2} = \lim_{x \rightarrow 2} (x^2 + 2x + 2) = 10 \end{aligned}$$

$$\text{(b) } y - 4 = 10(x - 2) \text{ or } y = 10x - 16$$



29. (a) Estimating $f'(1)$ from the triangle in the graph,

$$\text{we get } \frac{\Delta y}{\Delta x} \approx \frac{-0.37}{0.50} = -0.74.$$

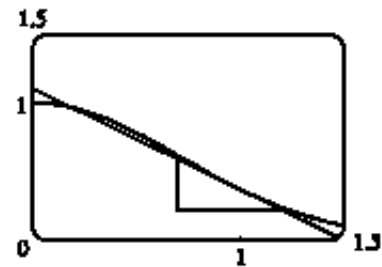
To estimate $f'(1)$ numerically, we have

$$f'(1) = \lim_{h \rightarrow 0} \frac{f(1+h) - f(1)}{h} = \lim_{h \rightarrow 0} \frac{e^{-(1+h)^2} - e^{-1}}{h} = y$$

From the table, we have $f'(1) \approx -0.736$.

(b) $y - e^{-1} \approx -0.736(x - 1)$ or $y \approx -0.736x + 1.104$

(c) See the graph in part (a).

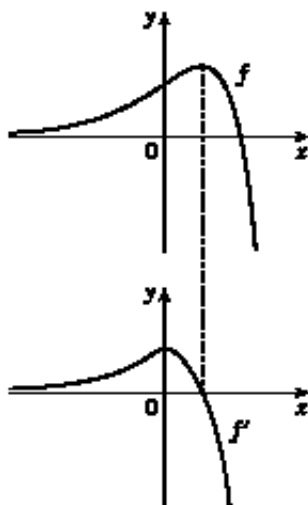


h	y
0.01	-0.732
0.001	-0.735
0.0001	-0.736
-0.01	-0.739
-0.001	-0.736
-0.0001	-0.736

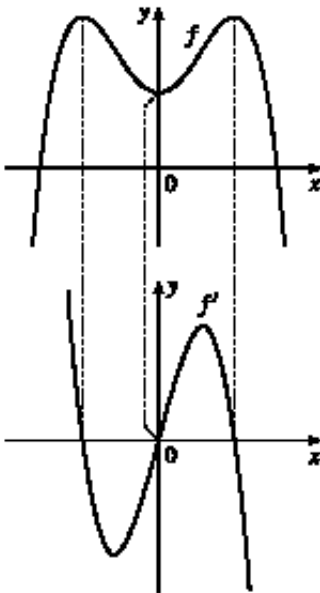
30. $2^6 = 64$, so $f(x) = x^6$ and $a = 2$.

31. (a) $f'(r)$ is the rate at which the total cost changes with respect to the interest rate. Its units are dollars/(percent per year).
 (b) The total cost of paying off the loan is increasing by \$1200/(percent per year) as the interest rate reaches 10%. So if the interest rate goes up from 10% to 11%, the cost goes up approximately \$1200.
 (c) As r increases, C increases. So $f'(r)$ will always be positive.

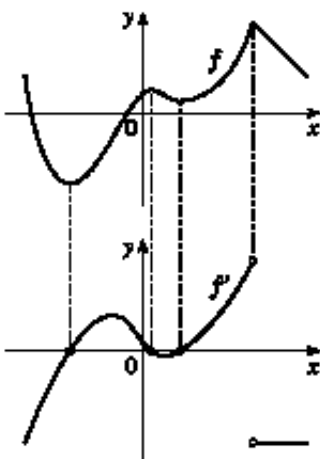
32.



33.



34.



$$35. (a) f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{\sqrt{3-5(x+h)} - \sqrt{3-5x}}{h} \frac{\sqrt{3-5(x+h)} + \sqrt{3-5x}}{\sqrt{3-5(x+h)} + \sqrt{3-5x}}$$

$$= \lim_{h \rightarrow 0} \frac{[3-5(x+h)] - (3-5x)}{h(\sqrt{3-5(x+h)} + \sqrt{3-5x})} = \lim_{h \rightarrow 0} \frac{-5}{\sqrt{3-5(x+h)} + \sqrt{3-5x}} = \frac{-5}{2\sqrt{3-5x}}$$

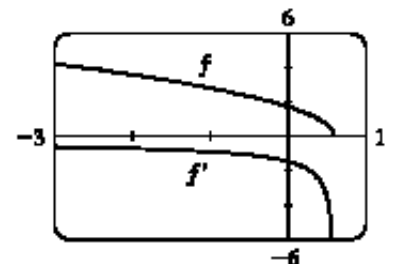
(b) Domain of f : (the radicand must be nonnegative) $3 - 5x \geq 0 \Rightarrow$

$$5x \leq 3 \Rightarrow x \in \left(-\infty, \frac{3}{5}\right]$$

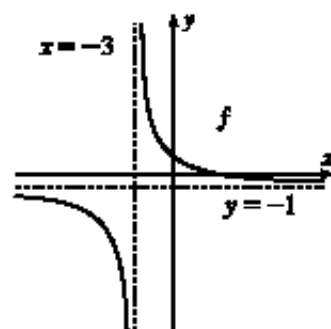
Domain of f' : exclude $\frac{3}{5}$ because it makes the denominator zero;

$$x \in \left(-\infty, \frac{3}{5}\right)$$

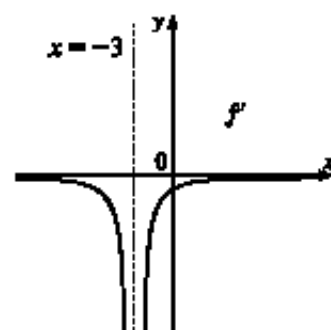
(c) Our answer to part (a) is reasonable because $f'(x)$ is always negative and f is always decreasing.



36. (a) As $x \rightarrow \pm\infty$, $f(x) = (4 - x)/(3 + x) \rightarrow -1$, so there is a horizontal asymptote at $y = -1$. As $x \rightarrow -3^+$, $f(x) \rightarrow \infty$, and as $x \rightarrow -3^-$, $f(x) \rightarrow -\infty$. Thus, there is a vertical asymptote at $x = -3$.



- (b) Note that f is decreasing on $(-\infty, -3)$ and $(-3, \infty)$, so f' is negative on those intervals. As $x \rightarrow \pm\infty$, $f' \rightarrow 0$. As $x \rightarrow -3^-$ and as $x \rightarrow -3^+$, $f' \rightarrow -\infty$.



$$\begin{aligned}
 \text{(c) } f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{\frac{4 - (x+h)}{3 + (x+h)} - \frac{4 - x}{3 + x}}{h} \\
 &= \lim_{h \rightarrow 0} \frac{(3+x)[4 - (x+h)] - (4-x)[3 + (x+h)]}{h[3 + (x+h)](3+x)} \\
 &= \lim_{h \rightarrow 0} \frac{(12 - 3x - 3h + 4x - x^2 - hx) - (12 + 4x + 4h - 3x - x^2 - hx)}{h[3 + (x+h)](3+x)} \\
 &= \lim_{h \rightarrow 0} \frac{-7h}{h[3 + (x+h)](3+x)} = \lim_{h \rightarrow 0} \frac{-7}{[3 + (x+h)](3+x)} = -\frac{7}{(3+x)^2}
 \end{aligned}$$

- (d) The graphing device confirms our graph in part (b).

37. f is not differentiable: at $x = -4$ because f is not continuous, at $x = -1$ because f has a corner, at $x = 2$ because f is not continuous, and at $x = 5$ because f has a vertical tangent.
38. The graph of a has tangent lines with positive slope for $x < 0$ and negative slope for $x > 0$, and the values of c fit this pattern, so c must be the graph of the derivative of the function for a . The graph of c has horizontal tangent lines to the left and right of the x -axis and b has zeros at these points. Hence, b is the graph of the derivative of the function for c . Therefore, a is the graph of f , c is the graph of f' , and b is the graph of f'' .

39. $C'(1990)$ is the rate at which the total value of U.S. currency in circulation is changing in billions of dollars per year. To estimate the value of $C'(1990)$, we will average the difference quotients obtained using the times $t = 1985$ and $t = 1995$.

$$\text{Let } A = \frac{C(1985) - C(1990)}{1985 - 1990} = \frac{187.3 - 271.9}{-5} = \frac{-84.6}{-5} = 16.92 \text{ and}$$

$$B = \frac{C(1995) - C(1990)}{1995 - 1990} = \frac{409.3 - 271.9}{5} = \frac{137.4}{5} = 27.48. \text{ Then}$$

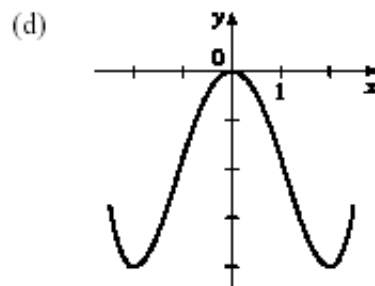
$$C'(1990) = \lim_{t \rightarrow 1990} \frac{C(t) - C(1990)}{t - 1990} \approx \frac{A + B}{2} = \frac{16.92 + 27.48}{2} = \frac{44.4}{2} = 22.2 \text{ billion dollars/year.}$$

40. Let $C(t)$ be the function that denotes the cost of living in terms of time t . $C(t)$ is an increasing function, so $C'(t) > 0$. Since the cost of living is rising at a slower rate, the slopes of the tangent lines are positive but decreasing as t increases. Hence, $C''(t) < 0$.

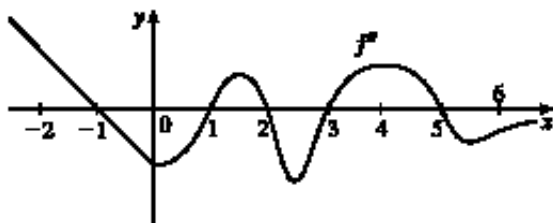
41. (a) $f'(x) > 0$ on $(-2, 0)$ and $(2, \infty) \Rightarrow f$ is increasing on those intervals. $f'(x) < 0$ on $(-\infty, -2)$ and $(0, 2) \Rightarrow f$ is decreasing on those intervals.

- (b) $f'(x) = 0$ at $x = -2, 0,$ and 2 , so these are where local maxima or minima will occur. At $x = \pm 2$, f' changes from negative to positive, so f has local minima at those values. At $x = 0$, f' changes from positive to negative, so f has a local maximum there.

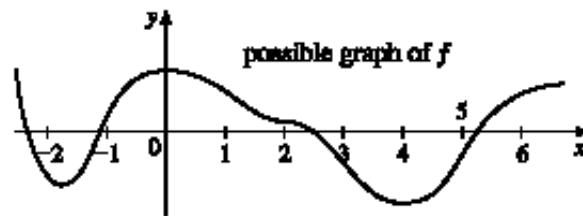
- (c) f' is increasing on $(-\infty, -1)$ and $(1, \infty) \Rightarrow f'' > 0$ and f is concave upward on those intervals. f' is decreasing on $(-1, 1) \Rightarrow f'' < 0$ and f is concave downward on this interval.



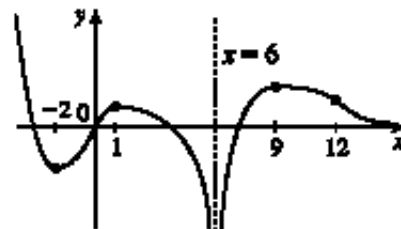
42. (a)



(b)



43. $f(0) = 0$, $f'(-2) = f'(1) = f'(9) = 0$, $\lim_{x \rightarrow \infty} f(x) = 0$, $\lim_{x \rightarrow 6} f(x) = -\infty$,
 $f'(x) < 0$ on $(-\infty, -2)$, $(1, 6)$, and $(9, \infty)$, $f'(x) > 0$ on $(-2, 1)$ and $(6, 9)$,
 $f''(x) > 0$ on $(-\infty, 0)$ and $(12, \infty)$, $f''(x) < 0$ on $(0, 6)$ and $(6, 12)$

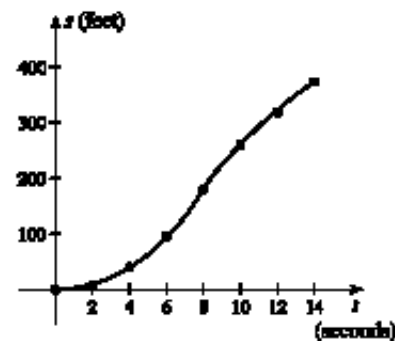


44. (a) Drawing slope triangles, we obtain the following estimates: $F'(1950) \approx \frac{1.1}{10} = 0.11$, $F'(1965) \approx \frac{-1.6}{10} = -0.16$, and $F'(1987) \approx \frac{0.2}{10} = 0.02$.
- (b) The rate of change of the average number of children born to each woman was increasing by 0.11 in 1950, decreasing by 0.16 in 1965, and increasing by 0.02 in 1987.
- (c) There are many possible reasons:
- In the baby-boom era (post-WWII), there was optimism about the economy and family size was rising.
 - In the baby-bust era, there was less economic optimism, and it was considered less socially responsible to have a large family.
 - In the baby-boomlet era, there was increased economic optimism and a return to more conservative attitudes.

45. (a) Using the data closest to $t = 6$, we have $\frac{s(8) - s(6)}{8 - 6} = \frac{180 - 95}{2} = 42.5$ and $\frac{s(4) - s(6)}{4 - 6} = \frac{40 - 95}{-2} = 27.5$. Averaging these two values gives us $\frac{42.5 + 27.5}{2} = 35$ ft/s as an estimate for the speed of the car after 6 seconds.

(b) From the graph, it appears that the inflection point is at $(8, 180)$.

(c) The velocity of the car is at a maximum at the inflection point.



46. Let f be the function shown. Since f is negative for $x < 0$ and positive for $x > 0$, F is decreasing for $x < 0$ and increasing for $x > 0$. f is increasing on $(-a, a)$ (from the low point to the high point) so its derivative f' (the second derivative of F) is positive, making F concave upward on $(-a, a)$. f is decreasing elsewhere, so its derivative f' is negative and F is concave downward on $(-\infty, -a)$ and (a, ∞) .

