Math 1a: Chapter 5 Review

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Section 4.9: Antiderivatives

- A function F is called an **antiderivative** of f on an interval I if $F'(x) = f(x)$ for all x in I.
- The general antiderivative is $F(x) + C$, where C is an arbitrary constant.
- The graph of the antiderivative F follows the **direction field**, given by little line segments of slope $f(x)$.

Section 5.1: Areas and Distances

- The **area** A of a region S that lies under the graph of the continuous function f is the limit of the sum of the areas of approximating rectangles:

$$A = \lim_{n\to\infty} R_n = \lim_{n\to\infty}\left[f(x_1)\Delta x + ... + f(x_n)\Delta x_n\right] = \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\Delta x$$

- The **distance traveled** is the area under the graph of the velocity function.
- Note that we consider the area under the *x*-axis as "negative" area.

Section 5.2: The Definite Integral

- A **Riemann Sum** is a sum of the form $\sum_{i=1}^{n} f(x_i^*)\Delta x$
- **Definition of a Definite Integral.** Let f be a continuous function defined for $a \le x \le b$. Divide the interval $[a,b]$ into n subintervals of width $\Delta x = (b-a)/n$. Let $x_0, x_1, ...x_n$ be the endpoints of those intervals, with $x_0 = a$ and $x_n = b$. Let $x_1^*, ..., x_n^*$ be sample points in these subintervals. The **definite integral from *a* to *b*** is $\int_a^b f(x)dx = \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i^*)\Delta x$.
- A definite integral gives the **net area** under a curve
- **Helpful equations for evaluating definite integrals using Riemann Sums:**

$$\sum_{i=1}^{n} c = nc \qquad \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} ca_i = c\sum_{i=1}^{n} a_i \qquad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{i=1}^{n}(a_i + b_i) = \sum_{i=1}^{n} a_i + \sum_{i=1}^{n} b_i \qquad \sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2$$

$$\sum_{i=1}^{n}(a_i - b_i) = \sum_{i=1}^{n} a_i - \sum_{i=1}^{n} b_i$$

- The **midpoint rule** says we can approximate a definite integral by evaluating the Riemann sum at the midpoints of the intervals
- **Properties of the Definite Integral:**

  $\int_b^a f(x)dx = -\int_a^b f(x)dx$ $\qquad$ $\int_a^a f(x)dx = 0$

  $\int_a^b c\,dx = c(b-a)$, where c is any constant

  $\int_a^b cf(x)dx = c\int_a^b f(x)dx$, where c is any constant

  $\int_a^b [f(x)-g(x)]dx = \int_a^b f(x)dx - \int_a^b g(x)dx$

  $\int_a^b [f(x)+g(x)]dx = \int_a^b f(x)dx + \int_a^b g(x)dx$

  If $f(x) \geq 0$ for $a \leq x \leq b$, then $\int_a^b f(x)dx \geq 0$

  If $f(x) \geq g(x)$ for $a \leq x \leq b$, then $\int_a^b f(x)dx \geq \int_a^b g(x)dx$

  If $m \leq f(x) \leq M$ for $a \leq x \leq b$, then $m(b-a) \leq \int_a^b f(x)dx \leq M(b-a)$

Section 5.3: Evaluating Definite Integrals

- **Evaluation Theorem.** If *f* is continuous on $[a,b]$, then $\int_a^b f(x)dx = F(b)-F(a)$, where *F* is any antiderivative of *f*.
- **Indefinite Integral.** Note that a definite integral gives a number, where an indefinite integral gives a function. $\int f(x)dx = F(x)$ means $F'(x) = f(x)$.
- **Table of Indefinite Integrals:** see page 369 of the textbook. Remember, finding indefinite integrals is just like finding antiderivatives.
- **Net Change Theorem:** The integral of a rate of change is the net change:

  $\int_a^b F'(x)dx = F(b)-F(a)$

Section 5.4: The Fundamental Theorem of Calculus

- **The Fundamental Theorem of Calculus.** Suppose *f* is continuous on $[a,b]$.

  1. If $g(x) = \int_a^x f(t)dt$, then $g'(x) = f(x)$. Thus, $\frac{d}{dx}\int_a^x f(t)dt = f(x)$.

  2. $\int_a^b f(x)dx = F(b)-F(a)$, where $F' = f$.

Section 5.5: The Substitution Rule

- **The Substitution Rule.** If $u = g(x)$ is a differentiable function whose range is an interval *I*, and f is continuous on *I*, then $\int f(g(x))g'(x)dx = \int f(u)du$.
- **The Substitution Rule for Definite Integrals.** If g' is continuous on $[a,b]$ and *f* is continuous on the range of $u = g(x)$, then $\int_a^b f(g(x))g'(x)dx = \int_{g(a)}^{g(b)} f(u)du$.
- **Integrals of Symmetric Functions.** Suppose *f* is continuous on $[-a,a]$.
  1. If *f* is even $\left[f(-x) = f(x)\right]$, then $\int_{-a}^{a} f(x)dx = 2\int_0^a f(x)dx$.
  2. If *f* is odd $\left[f(-x) = -f(x)\right]$, then $\int_{-a}^{a} f(x)dx = 0$.