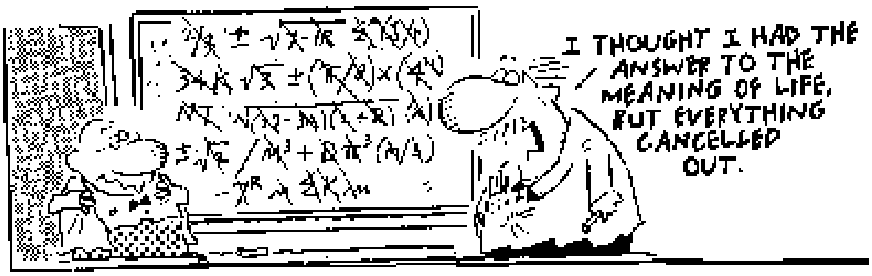
Iain and Jean's Math 1a Final Exam Study Sheet
Reading period 2006



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Knowledge
- Basic trig
- Derivatives of trig and exponential functions, their related integrals
- Formula for tangent line
- Squeeze Theorem, Intermediate Value Theorem, Mean Value Theorem

Skills
- Limits
  - Use limit laws to solve limit problems of all kinds
    - Find constants that satisfy given limits
  - Use Squeeze Theorem to find limits
  - Graphs
    - Identify whether something is continuous, or identify discontinuities
    - Identify asymptotes and intercepts
    - Increasing/decreasing, concave up/down
    - Applications of Intermediate Value Theorem
  - Solve problems using L'Hospital's Rule—know when and when not to use
- Derivatives
  - Estimate equation of tangent line using secant lines
  - Draw velocity and acceleration graphs from position graph (analogous: find equations)
  - Given definition of derivative, identify point at which we are approximating slope of tangent line
  - Use tangent lines for linear approximations
  - Relative error, percentage error
  - Given f(x), find f'(x) and f''(x)
    - Apply chain rule, power rule, product rule, and quotient rule
  - Implicit differentiation
  - Given function and roots, solve for constants to satisfy certain requirements
  - Related rates
  - Optimization
  - Logarithmic differentiation
- Integrals
  - Approximate integrals using Riemann sums with left/right endpoints and midpoints (likely from a graph or table)
  - Find and evaluate definite/indefinite integrals
  - Evaluate integrals using u-substitution
- Other things to understand
  - Absolute value
  - Inverses

Limits

$\lim_{x\to a} f(x)$ $\quad \lim_{x\to a} x^2 = a^2$

- straight forward problems – can just plug it in
- removable discontinuity
- limit laws
- L'Hospital's Rule → graph

Ex: $\lim_{x\to 0} (1+\sin x)^{\cot x} \to 1^\infty$

let $y = (1+\sin x)^{\cot x}$

$\ln y = \cot x[\ln(1+\sin x)]$

$\lim_{x\to 0} \ln y = \lim_{x\to 0} \cot x(\ln(1+\sin x))$

$= \lim_{x\to 0} \frac{\ln(1+\sin x)}{\tan x} \left(\frac{0}{0}\right)$

$= \lim_{x\to 0} \frac{\frac{\cos x}{1+\sin x}}{\sec^2 x} = \lim_{x\to \infty} \frac{\cos^3 x}{1+\sin x} = \boxed{1}$

$\lim_{x\to 0} (\ln y) = 1$

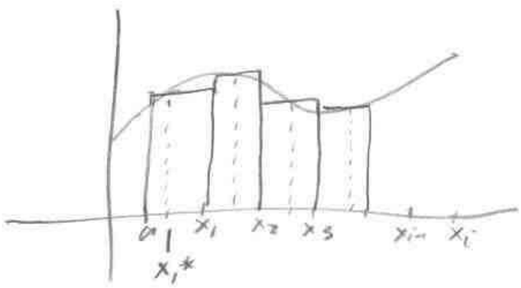
$\lim_{x\to 0} e^{\ln y} = e^1 \Rightarrow \lim_{x\to 0} y = e$

$\Rightarrow \boxed{\lim_{x\to 0} (1+\sin x)^{\cot x} = e}$

Riemann Sums

$$A = \lim_{n\to\infty} R_n = \lim_{n\to\infty} [f(x_1)\Delta x + f(x_2)\Delta x + \ldots + f(x_n)\Delta x]$$

$$A = \lim_{n\to\infty} L_n = \lim_{n\to\infty} [f(x_0)\Delta x + f(x_1)\Delta x + \ldots + f(x_{n-1})\Delta x]$$



underapprox.
vs. overapprox.

$$A = \lim_{n\to\infty} [f(x_1^*)\Delta x + f(x_2^*)\Delta x + \ldots + f(x_n^*)\Delta x]$$

Definite integral w/ sample pts.

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x$$

Midpoint Rule

$$\int_a^b f(x)\,dx \approx \sum_{i=1}^{n} f(\bar{x}_i)\Delta x = \Delta x [f(\bar{x}_1) + \ldots + f(\bar{x}_n)]$$

where $\Delta x = \frac{b-a}{n}$

$\bar{x}_i = \frac{1}{2}(x_{i-1} + x) =$ midpoint of $[x_{i-1}, x_i]$

Properties of the Integral

1. $\int_a^b c\,dx = c(b-a)$, where is any constant
2. $\int_a^b [f(x)+g(x)]\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$
3. $\int_a^b cf(x)\,dx = c\int_a^b f(x)\,dx$, where c is any constant
4. $\int_a^b [f(x)-g(x)]\,dx = \int_a^b f(x)\,dx - \int_a^b g(x)\,dx$

$$\left\{\begin{array}{l} 5.\ \int_a^b f(x) = -\int_b^a f(x)\,dx \\ 6.\ \int_a^a f(x) = 0 \\ 7.\ \int_a^c f(x)\,dx + \int_c^b f(x)\,dx = \int_a^b f(x)\,dx \end{array}\right.$$

✓8 inverse trig
know trig integrals

$\int x^n dx = \frac{x^{n+1}}{n+1}$

$\int \frac{1}{x} dx = \ln|x| + C$

$\int e^x dx = e^x + C$

$\int k f(x)\, dx = k \int f(x)\, dx$, where k is a constant

$\int (f(x) \pm g(x))\, dx = \int f(x)\, dx \pm \int g(x)\, dx$

Fundamental Theorem of Calculus (Part 1)

If f is continuous on $[a,b]$, then g is defined by

$$g(x) = \int_a^x f(t)\, dt \quad a \leq x \leq b$$

is an antiderivative of f, that is, $g'(x) = f(x)$ for $a < x < b$.

(Part 2)

$\int_a^b f(x)\, dx = F(b) - F(a)$, where F is any antiderivative of f, that is, $F' = f$.

f even $\rightarrow [f(-x) = f(x)]$, then $\int_{-a}^{a} f(x)\, dx = 2\int_0^a f(x)\, dx$

f odd $\rightarrow [f(-x) = -f(x)]$, then $\int_{-a}^{a} f(x)\, dx = 0$

u-substitution

If g' is continuous on $[a,b]$ and f is continuous on the range of $u = g(x)$, then

$$\int_a^b f(g(x))\, g'(x)\, dx = \int_{g(a)}^{g(b)} f(u)\, du$$

strategies: $\int f(x)\, dx$

- the derivative of u should be part of $f(x)$
- include constants in u
- sometimes you will have to simplify first
- remember to change the bounds of integration when doing u-subst. for definite integrals

Ex. $\int 5\sec 4x \tan 4x\,dx = 5\int \sec u \tan u \left(\frac{1}{4}\right)du$

let $u = 4x$ $\quad = \frac{5}{4}\int \sec u \tan u\,du$

$du = 4dx$

$\frac{1}{4}du = dx$ $\quad = \frac{5}{4}\sec u + C = \frac{5}{4}\sec 4x + C$

Ex. $\int (\sin x + \cos x)^2 dx = \int(\sin^2 x + \cos^2 x)dx + \int 2\sin x\cos x\,dx$

$= \int dx + 2\underbrace{\int \sin x \cos x\,dx}$

let $u = \sin x$

$du = \cos x$

$= x + 2\int u\,dx$

$= x + 2\cdot\left(\frac{1}{2}\right)u^2 + C$

Evaluate.

$= x + \sin^2 x + C$

Ex: $\int_3^5 \frac{\sqrt{x^2-9}}{x}\,dx$

$\int \frac{\sqrt{x^2-9}}{x}\,dx = \int \frac{x\sqrt{x^2-9}}{x^2}\,dx$

$x^2 - 9 = u^2$

$x\,dx = u\,du$

$\int \frac{u^2}{u^2+9}\,du = \int \frac{u^2-9+9}{u^2+9}\,du = \int \frac{u^2+9}{u^2+9}\,du + \int \frac{9}{u^2+9}\,du$

$= \int du + 9\int \frac{1}{u^2+9}\,du$

$\frac{d}{dx}[\tan^{-1}x] = \frac{1}{1+x^2}$ $\quad = u - 3\tan^{-1}\left(\frac{u}{3}\right)$

$= \sqrt{x^2-9} - 3\tan^{-1}\frac{\sqrt{x^2-9}}{3}$

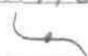
Answer $= \boxed{1.218}$

Ex: $\int \frac{\sin(\ln x + 5)}{x}\,dx = \int \sin(\ln x + 5)\frac{1}{x}dx = \int \sin u\,du$

let $u = \ln x + 5$ $\quad = -\cos u + C$

$du = \frac{1}{x}dx$ $\quad = -\cos(\ln x + 5) + C$

sometimes / always / never

- A continuous function is differentiable.
  → sometimes
- A diff. function is continuous.
  → always
- If $\lim_{x \to a} f(x) = f(a)$, then function is cont.
  → always
- If you can draw a curve w/out lifting your pencil, then it's a cont. function.
  → sometimes
- Have critical pt. x, the $f'(x) = 0$.
  → sometimes
- If $f'(x) = 0$ and x is a pt. of inflection, then graph looks like  or 
  → always

Economics problem

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A manufacturer has been selling 1000 TV sets per week at \$450 each. with \$10 rebate, it sells 100 more.

a) Find demand

x	$P(x)$
1000	\$450
1100	\$440

we see that it is a line, so we need to find formula

$y = mx + b$

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$\frac{440-450}{1100-1000} = -\frac{10}{100} = -\frac{1}{10}$

$f(x) = -\frac{1}{10}x + b$

$f(x) = -\frac{1}{10}x + 550$

b) How large of a rebate should be offered to maximize revenue?

$rev = P(x)x$

$= -\frac{1}{10}x^2 + 550x$

Remember to verify max!

Take derivative to find maximum

c) cost $C(x) = 68000 + 150x$

Find max. profit.

profit = rev. − cost

$P(x) = -\frac{1}{10}x^2 + 550x - (68000 - 150x)$

$P(x) = -\frac{1}{5}x + 400$

$0 = -\frac{1}{5}x + 400$

$400 = \frac{1}{5}x$

$x = 2000$

$P'(x)$ + | − max!

2000