Math 1a. §4.6 Worksheet
Optimization Problems

Fall 2005

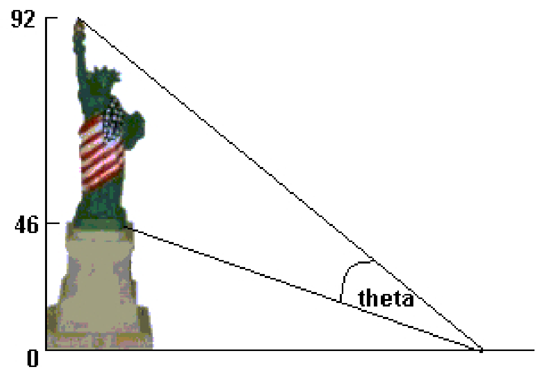
Strategy for Optimization Problems

1. Draw the picture.
2. Name the variables.
3. Write down the function that needs to be maximized or minimized.
4. Write down the relationships between the variables.
5. Reduce the function that is to be optimized to a function in one variable using the relationships in the previous step.
6. Find the critical values of the function.
7. Apply the First or Second Derivative Test to the Function. Don't forget to test the endpoints.
8. Write down the final answer.

Optimization Problems

1. You are in a row boat 2 miles from a straight shoreline. Six miles down the shoreline from the nearest point on shore is an outhouse. If you can row at 2 mph and run at 6 mph, what point along the shoreline should you aim in order to minimize the amount of time it will take to get to the outhouse?

2. To get the best view of the Statue of Liberty, you should be at the position where θ is at a maximum. If the statue stands 92 meters high, including the pedestal, which is 46 meters high, how far from the base should you be? [*Hint*: Find a formula for θ in terms of your distance from the base. Use this function to maximize θ, noting that $0 < \theta < \pi/2$.]



3. A Norman window has the shape of a rectangle surmounted by a semicircle. Thus, the diameter of the semicircle is equal to the width of the rectangle. If the perimeter of the window is 30 ft, find the dimensions of the window so that the greatest amount of light is admitted.

4. A Christmas ornament is to be constructed by inscribing a right circular cone of brightly colored material in a transparent spherical ball of radius 2 inches. What is the maximum possible volume of such a cone?

5. You have just invented a new peanut butter guacamole dip, and you stand in front of the Science Center to sell your product by the jar. Somehow, a rumor gets started (not traceable to you) that your dip cures acne, baldness, and acts as a sedative for younger siblings. Sales take off. At a price of \$1.00 per jar, you can sell 500 jars a day. For every nickel that you increase the price, you sell two fewer jars. Assuming that your fixed costs are \$200 per day (protection money), and the cost per jar is \$0.50, determine the price for which you should sell your dip in order to maximize profit.

6. A plot of ground in the shape of a circular sector (a wedge of pie) is to have a border of roses along the straight lines and tulips along the circular arc. Roses cost \$20 per meter and tulips cost \$15 per meter. If the area of the plot is to be 100 square meters, what is the least the flowers can cost? [*Hint*: The area of a sector is given by $A = r^2\theta/2$, and the length of a sector is given by $L = r\theta$, where θ is the angle of the sector in radians.]