Name: ______________________________ ID#: ____________________

Final Exam

Math 1a
Introduction to Calculus

21 January 2005

Show all of your work. Full credit may not be given for an answer alone. You may use the backs of the pages or the extra pages for scratch work. Do not unstaple or remove pages.

This is a non-calculator exam.

Please check your section:

☐	1.0	MWF10	Tatyana Chmutova	☐	4.0	TΘ10	Dawei Chen
☐	1.1	MWF10	Matthew Leingang	☐	4.1	TΘ10	Jerrel Mast
☐	2.0	MWF11	Ethan Cotterill	☐	4.2	TΘ10	Chun-Chun Wu
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				☐	5.1	TΘ11:30	Sonal Jain

Students who, for whatever reason, submit work not their own will ordinarily be required to withdraw from the College.

—*Handbook for Students*

Problem Number	Possible Points	Points Earned
1	18	
2	8	
3	20	
4	8	
5	15	
6	18	
7	18	
8	8	
9	8	
10	15	
11	14	
Total	150	

1. (18 Points) Compute the following limits, with justification.

(i) $\lim_{x\to\infty} \frac{\sqrt{x^2+1}}{x^2+7}$

(ii) $\lim_{x\to\pi/2^+} \tan(x)$

(iii) $\lim_{x\to 1} \frac{x-1}{e^x-1}$

(iv) $\lim_{x\to 0} \frac{2\sec(x)-2-x^2}{x^2}$

2. (8 Points) Let f be the function $f(x) = \sin(x) - \frac{x}{3}$.

(a) Use the Intermediate Value Theorem to show that there exists a point c in $\left(\frac{\pi}{2}, \pi\right)$ such that $f(c) = 0$.

(b) Suppose that there was another point d in the same interval such that $f(d) = 0$. What fact would the Mean Value Theorem allow you to conclude? Why is this "fact" *impossible*?

Once you have shown this, you will have proven that c is the unique solution in the interval to the equation $f(x) = 0$.

_____ / 8

3. (20 Points) Find the following derivatives.

(i) $\dfrac{d}{dx}(3x^2 + 4x + 6)$

(ii) $\dfrac{d}{dx}\dfrac{2^x}{1 + \ln x}$

(iii) $\frac{d}{dx}\cos\left(x^{1/3}\right)$

(iv) $\frac{d}{dx}x^{\sqrt{x}}$

______ / 20

4. (8 Points)

(a) Write down the function which is the linear approximation to the square root function at $a = \frac{9}{4}$.

(b) Use this function to approximate $\sqrt{2}$.

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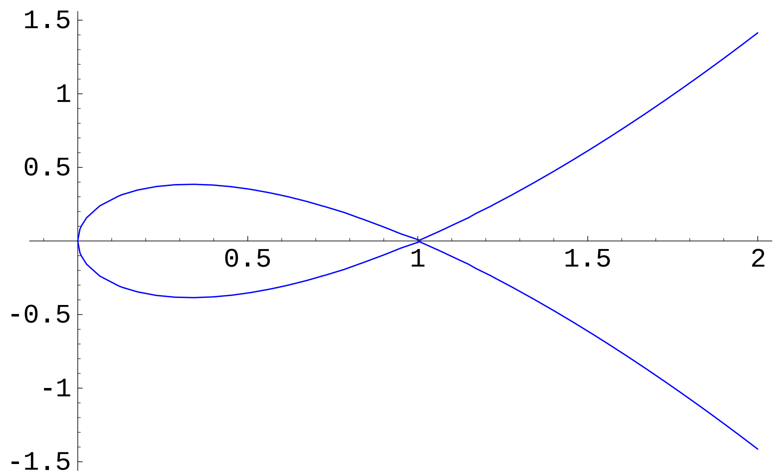
5. (15 Points) The relation $y^2 = x(x-1)^2$ defines a curve in the plane.



(i) Solve for y in terms of x, and use it to find $\dfrac{dy}{dx}$ at the point $\left(\frac{1}{4}, -\frac{3}{8}\right)$.

(ii) Find $\frac{dy}{dx}$ implicitly in terms of y and x. What is its value at $\left(\frac{1}{4}, -\frac{3}{8}\right)$?

(iii) A parametrization of the curve is given by

$$x(t) = t^2 \qquad\qquad y(t) = t(t^2 - 1).$$

Find $\frac{dy}{dx}$ in terms of t. What is its value at $t = \frac{1}{2}$? (This corresponds to the point $\left(\frac{1}{4}, -\frac{3}{8}\right)$).

______ / 15

6. (18 Points) **The dreaded graphing problem.** Let

$$f(x) = \frac{1}{x+1} - \frac{1}{(x+1)^2}.$$

(a) Find all horizontal and vertical asymptotes of f.

(b) The derivative of f is

$$f'(x) = -\frac{1}{(x+1)^2} + \frac{2}{(x+1)^3}.$$

Find the intervals of increase or decrease.

Increasing on : ____________________

Decreasing on : ____________________

(c) Find any local maxima or minima.

(d) The second derivative of f is

$$f''(x) = \frac{2}{(x+1)^3} - \frac{6}{(x+1)^4}$$

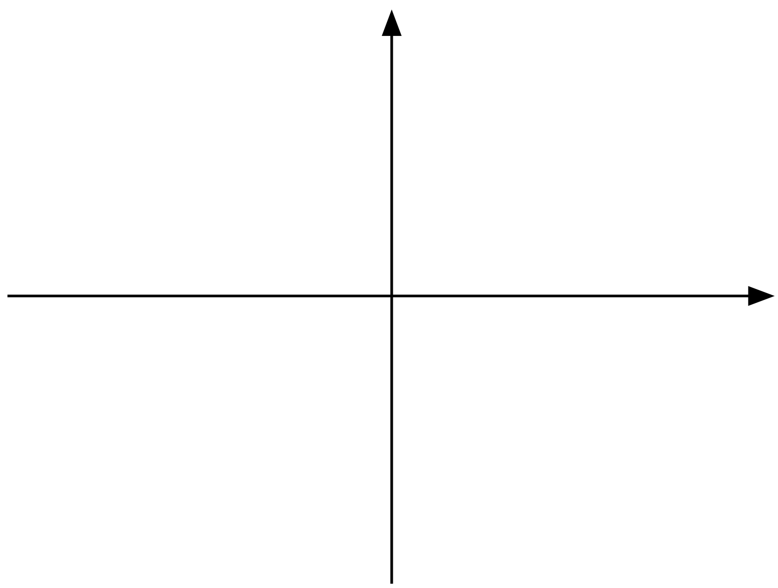
Find the intervals of concavity.

Concave up on : ______________________________

Concave down on : ______________________________

(e) Find any inflection point(s).

(f) Sketch the graph of f. Label all the significant points you have found previously.



(g) Find the global minimum and maximum, if they exist.

______ / 18

7. (18 Points) Apex Corporation is planning to sell sponges on television. The sponges cost \$2 per package. Stacy believes that if they set the selling price at \$20 per package, they will sell 1000 packages, and for every dollar they increase the price, the quantity they will sell will decrease by 50 packages.

We will find the price at which profit will be maximized.

(a) Assuming Stacy's assumptions about the market are true, show that the demand curve (price in terms of quantity sold) is given by

$$p(x) = 40 - \tfrac{1}{50}x.$$

How many packages will be sold if the price is set at \$27?

(b) Now show that the profit (this is revenue minus costs, remember) is given by

$$K(x) = -\tfrac{1}{50}x(x - 1900)$$

(c) What price maximizes profit? Make sure you show it's maximal and not minimal!

______ / 18

8. (8 Points) Ferdbert Freshman is studying for his Math 1a Final. He starts studying at midnight and does problems at the rate of

$$r(t) = \frac{60}{\pi(t^2+1)}$$

problems per hour, where t is measured in hours after midnight. How many problems has he done by 1:00AM?

_____ / 8

9. (8 Points) Evaluate the following definite integrals.

(i) $\int_1^4 (2x + x^2)\,dx$

(ii) $\int_1^{e^{17}} \frac{1}{x}\,dx$

______ / 8

10. (15 Points) Compute the following integrals. For definite integrals, your answer should be a number. For indefinite integrals, your answer should be the most general antiderivative as a function of x.

(i) $\displaystyle\int \left(3x\sqrt{x^2+1}\right)\,dx$

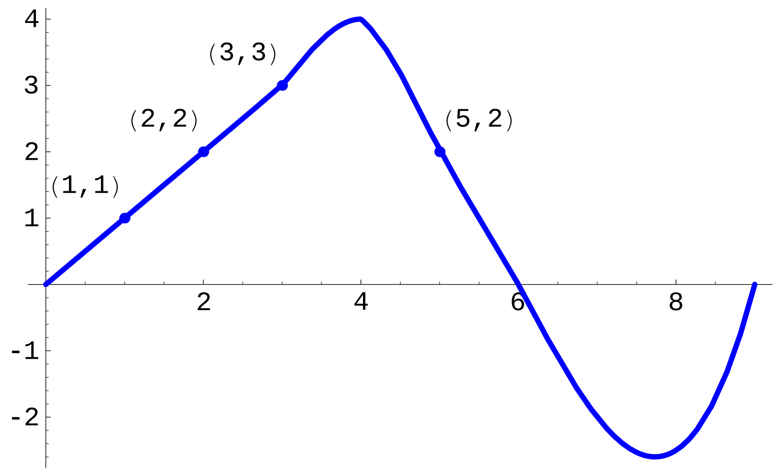
(ii) $\displaystyle\int \frac{\ln x}{x}\,dx$

10 **10**

(iii) $\int_0^1 \frac{e^{4x}}{1+e^{4x}}\,dx$

______ / 15

11. (14 Points) Suppose that f is the differentiable function shown in the graph below



(The function is a straight line from $(0,0)$ to $(3,3)$, and is differentiable at $x = 4$, even though the graph looks a little pointy.) Suppose the the position at time t seconds of a particle moving along a coordinate axis is

$$s(t) = \int_0^t f(x)\,dx$$

meters. Use the graph to answer the following questions. *Give reasons for your answers.*

(i) What is the particle's velocity at time $t = 5$?

(ii) Is the acceleration of the particle at time $t = 5$ positive or negative?

(iii) What is the particle's position at time $t = 3$?

(iv) At what time during the first 9 seconds does s have its largest value?

(v) Approximately when is the acceleration zero?

(vi) When is the particle moving toward the origin? Away from the origin?

(vii) On which side (positive or negative) of the origin does the particle lie at time $t = 9$?

______ / 14