Math 1, 2002 Midterm

Problems # 1, 2 by Peter Zoogman

1a) $\frac{d}{dx}(x^3 \sec x \ln x) = 3x^2 \sec x \ln x + x^3 \sec x \tan x \ln x + x^3 \sec x \cdot \frac{1}{x}$

$= x^2 \sec x (3 \ln x + x \tan x \ln x + 1)$

b) $\frac{d}{dx} \sin e^{x^2} = \cos e^{x^2} \cdot \frac{d}{dx} e^{x^2}$

$= \cos e^{x^2} \cdot 2x e^{x^2}$

c) $\frac{d}{dx}\left(x \log_3 x + \frac{3^x}{x}\right) = \log_3 x + x \cdot \frac{1}{x \ln 3} + \frac{x 3^x \ln 3 - 3^x}{x^2}$

$= \log_3 x + \frac{1}{\ln 3} + \frac{3^x (x \ln 3 - 1)}{x^2}$

d) $\frac{d}{dx}(\tan^{-1}(3x^2)) = \frac{1}{1+(3x^2)^2} \cdot 6x = \frac{6x}{1+9x^4}$

e) $\frac{d}{dx}(4(\tan x)^5) = 4 \frac{d}{dx} \tan^5 x = 20 \tan^4 x \cdot \sec^2 x$

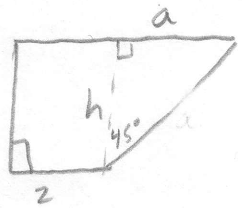
f) $\frac{d}{dx}(\cos(x+\cos x)) = -\sin(x+\cos x) \cdot (1 - \sin x)$

g) $(2+\cos x)^x = y \Rightarrow x \ln(2+\cos x) = \ln y$

$\ln(2+\cos x) + \frac{x}{2+\cos x} \cdot (-\sin x) = \frac{1}{y} y'$

$\frac{dy}{dx} = (2+\cos x)^x \left[\ln(2+\cos x) - \frac{x \sin x}{2+\cos x}\right]$

2a)



$a = h$

For area, we can add the areas of the square and the triangle to find the area of the trapezoid:

$$A(h) = 2h + \tfrac{1}{2}h \cdot h$$
$$= 2h + \tfrac{1}{2}h^2$$

b) given $\frac{dA}{dt} = -1 \text{ ft}^2/\text{min}$

by chain rule $\frac{dA}{dt} = \frac{dA}{dh} \cdot \frac{dh}{dt}$ (1)

and from part (a) we see $\frac{dA}{dh} = 2 + h$ (2)

so $\frac{dh}{dt} = \frac{-1}{2+h}$

and at $h=2$, $\frac{dh}{dt} = \frac{-1}{4}$ ft/min

c) given $\frac{dh}{dt} = 1$ ft/min

we still know (1) and (2) from above, so we see $\frac{dA}{dt} = (2+h)(1)$

So when $h=2$, $\frac{dA}{dt} = 4 \text{ ft}^2/\text{min}$