Math 1a - Exam 1 Review Problems - Fall 98

1) Find the following derivatives:

a) $g(t) = \tan t \ \ln(2 + e^{3t})$	b) $f(x) = \tan^{-1}(\frac{x-1}{x+1})$
c) $f(x) = \cos^2(\frac{3}{x})$	d) $k(x) = \lvert x^2 - 9 \rvert$ (Hint: Rewrite $k(x)$ according to how it is defined on those intervals on which the expression x^2 - 9 is positive or negative. Graph both $k(x)$ and $k'(x)$.)
e) Find $f'(1)$ for $f(x) = \frac{3}{\sqrt[3]{x}} - x - \frac{1}{x}$	f) Find $g'(\frac{p}{2})$ for $g(x) = \frac{x^2}{\cos(\cos x)}$

2) Suppose that all we know about a continuous, differentiable function $f(x)$ is the following numerical data:

x	0.2	0.4	0.6	0.8	1.0
$f(x)$	20	25	27	21	15

Give <u>estimates</u> for $f'(0.6)$ and $f''(0.6)$.

3) Consider the function $f(x) = \begin{Bmatrix} x^2 \sin(\frac{1}{x}) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{Bmatrix}$.

a) What is $f'(x)$ for $x \neq 0$?

b) What is $f'(0)$?

4) Bob and Carol and Ted and Alice were arguing about the quotient rule.

Bob said: $\left(\frac{f}{g}\right)' = \frac{(f \cdot g)' - 2f \cdot g'}{g^2}$

Carol said: $\left(\frac{f}{g}\right)' = f' \cdot \left(\frac{1}{g}\right)'$

Ted said: $\left(\frac{f}{g}\right)' = \lim_{h \to 0} \frac{\left(\frac{f}{g}\right)(x+h) - \left(\frac{f}{g}\right)(x)}{g^2}$

Alice said: $\left(\frac{f}{g}\right)' = \frac{f' - \frac{f \cdot g'}{g}}{g}$

Were any of them right? Why? (Here $\left(\frac{f}{g}\right)(x)$ denotes $\frac{f(x)}{g(x)}$.)

5) Which of the following expressions represent the derivative of f at $x = a$?:

a) $\lim_{h \to 0} \frac{f(a+2h) - f(a)}{2h}$

b) $\lim_{x \to a} \frac{f(x)}{x}$

c) $\lim_{h \to 0} \frac{f(a+h) - f(a-h)}{2h}$

d) $\lim_{x \to a} \frac{f(x) - f(a)}{x - a}$

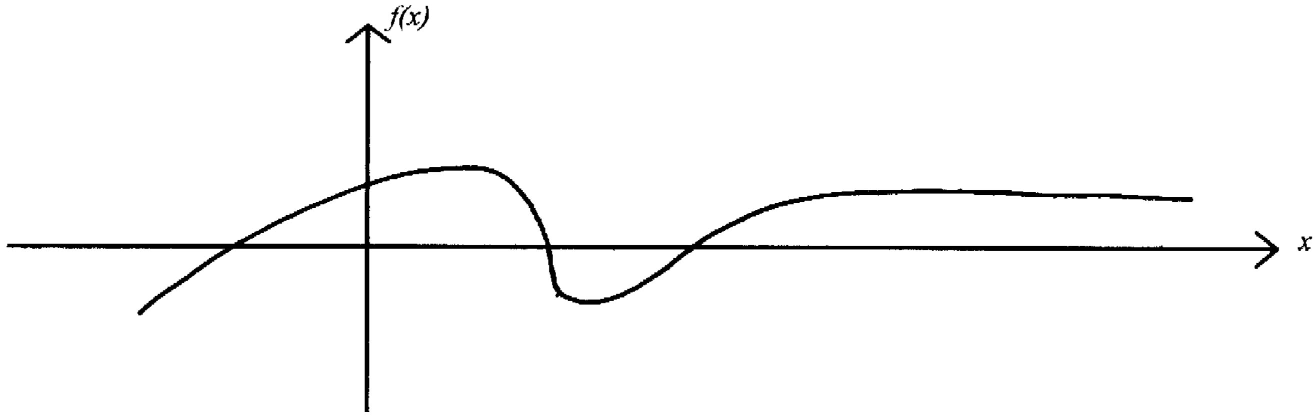
e) $\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$

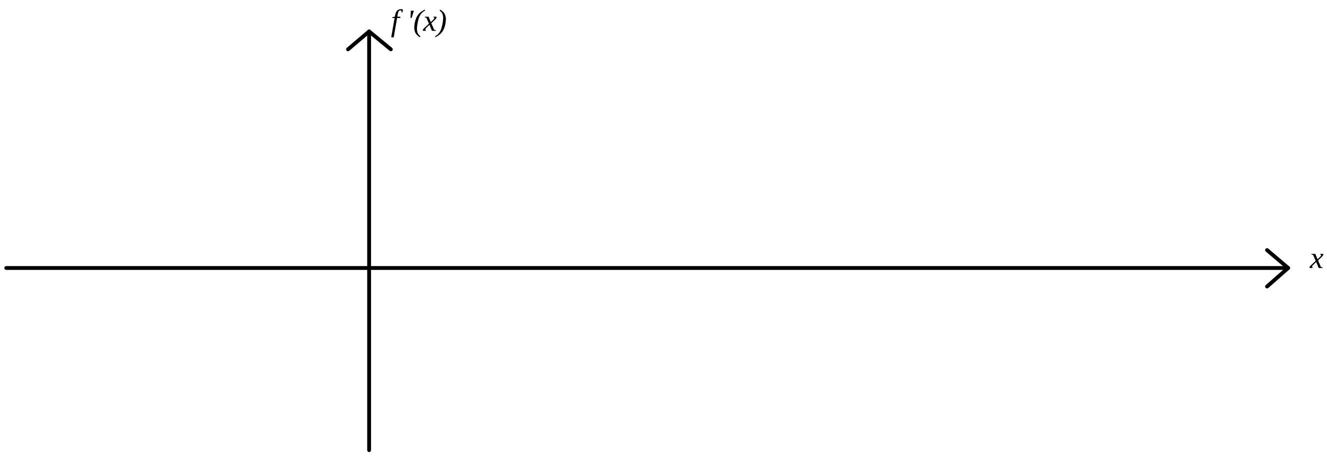
6) Use <u>the definition of the derivative</u> to compute $f'(0)$ for $f(x) = \frac{1}{2x+1}$.

7) A conical paper cup 8 inches across the top and 6 inches deep is full of water. The cup springs a leak at the bottom and loses water at the rate of 2 cubic inches per minute. How fast is the water level dropping at the instant when the water is exactly 3 inches deep?

8) An airplane is maintaining a constant altitude of 5 miles above sea level and is traveling at 507 miles per hour due west of LaGuardia Airport. When the airplane passes over a building 12 miles west of the airport, how fast is the angle between the airplane and the airport changing? Please include units in your answer.

9) Given the graph of the function $y = f(x)$ as shown, graph its derivative on the given axes, marking all noteworthy points appropriately.





10) An object travels on a straight line. Its position on that line is given by $s(t) = \frac{2}{3}t^3 - 7t^2 + 20t + 8$, starting at $t = 0$. When its velocity is positive, it is moving to the right. When its velocity is negative, it is moving to the left.

a) Find a formula for its velocity at time *t*.
b) Find a formula for its acceleration at time *t*.
c) When is it moving to the right?
d) When is its velocity decreasing?
e) When does the object change direction?

11) Find the equation of the tangent line to the curve $x^2y + 2y^3 = 3x + 2y + 54$ at the point where $(x, y) = (2,3)$.

12) Find the equations for two lines through the origin that are tangent to the curve with equation:
$x^2 - 4x + y^2 + 3 = 0$.

13) Consider the parabola $y = x^2$. At what point on this parabola is the tangent line parallel to the line connecting the points (1,1) and (3,9) on this parabola?

14) Use linear approximation to estimate $\sqrt[5]{31}$.

15) Use a linear approximation for $\ln x$ near $x = 1$ to find $\lim_{x\to 1}\left(\frac{\ln x}{x^2 - 1}\right)$.