

HW #2 Solutions

$$2.1.23) \textcircled{a} \lim_{x \rightarrow +\infty} x \sin\left(\frac{1}{x}\right) = \lim_{\frac{1}{x} \rightarrow 0^+} \left(\frac{1}{x}\right) \sin\left(\frac{1}{x}\right) = \lim_{x \rightarrow 0^+} \frac{\sin x}{x}$$

$$\textcircled{b} \lim_{x \rightarrow +\infty} \frac{1-x}{1+x} = \lim_{\frac{1}{x} \rightarrow 0^+} \frac{1-\frac{1}{x}}{1+\frac{1}{x}} = \lim_{x \rightarrow 0^+} \frac{x(1-\frac{1}{x})}{x(1+\frac{1}{x})} = \lim_{x \rightarrow 0^+} \frac{x-1}{x+1}$$

$$\textcircled{c} \lim_{x \rightarrow -\infty} \left(1 + \frac{2}{x}\right)^x = \lim_{\frac{1}{x} \rightarrow 0^-} \left(1 + 2x\right)^{\frac{1}{x}} = \lim_{x \rightarrow 0^-} \left(1 + 2x\right)^{\frac{1}{x}}$$

$$2.1.28) f(x) = \frac{x - \sin x}{x^3}$$

\textcircled{a} It appears to be going toward 0.1666, or $\frac{1}{6}$

\textcircled{b} These data may lead you to believe that the limit is zero

\textcircled{c} Numerical data may be faulty, especially if your calculator carries an insufficient number of decimal places.

x	f(x)
0.1	0.16658335
0.01	0.16666583
0.001	0.166666658
0.0001	0.166666666
0.00001	0.166666666
0.0000001	0.166
0.000000001	0.166
10^{-10}	0.166

Using a conventional calculator, these would be zero

$$2.2.2) \textcircled{a} \lim_{x \rightarrow 2} f(x) + \lim_{x \rightarrow 2} g(x) = 0 + 0 = 0$$

$$\textcircled{b} \lim_{x \rightarrow 0} f(x) + \lim_{x \rightarrow 2} g(x) = \text{DNE} + 2 \rightarrow \text{DNE}$$

$$\textcircled{c} \lim_{x \rightarrow 0^+} f(x) + \lim_{x \rightarrow 0^+} g(x) = -2 + 2 = 0$$

$$\textcircled{d} \lim_{x \rightarrow 0^-} f(x) + \lim_{x \rightarrow 0^-} g(x) = 1 + 2 = 3$$

$$\textcircled{e} \lim_{x \rightarrow 2} \frac{f(x)}{1+g(x)} = \frac{\lim_{x \rightarrow 2} f(x)}{\lim_{x \rightarrow 2} g(x) + 1} = \frac{0}{1+0} = 0$$