3.3/3

$y = 3x^8 + 2x + 1$, $y' = 3 \cdot 8x^7 + 2$

$= \boxed{24x^7 + 2}$

3.3/5

$y = \pi^3 = $ a constant

$y' = \boxed{0}$

3.3/13

$f(x) = x^{-3} + \frac{1}{x^7} = x^{-3} + x^{-7}$

$f'(x) = \boxed{-3x^{-4} - 7x^{-8}}$

3.3/20

$f(x) = (x^5 + 2x)^2$

$f'(x) = 2(x^5 + 2x) \cdot \frac{d}{dx}(x^5 + 2x)$

$= \boxed{2(x^5 + 2x)(5x^4 + 2)}$

3.3 / 77

$$f(x) = \begin{cases} 3x^2, & x \le 1 \\ ax+b, & x > 1 \end{cases}$$

$$f'(x) = \begin{cases} 6x, & x \le 1 \\ a, & x > 1 \end{cases}$$

in order that $f'(1)$ be defined:

f must be continuous at 1

f' must be continuous at 1

	$3x^2 = ax+b$	$6x = a$
$x=1$	$3 = a+b$	$a = 6$

$3 = 6 + b$

$b = -3$

$$\boxed{(a, b) = (6, -3)}$$

Math 1a HW#5 (Solutions)

Problems – Sec. 3.3 : 18, 26, 36, 38, 42, 48, 58, 71

(18) Find $f'(x)$

$$f'(x)=\frac{d}{dx}\left[\left(\frac{1}{x}+\frac{1}{x^2}\right)(3x^3+27)\right]=\left(\frac{1}{x}+\frac{1}{x^2}\right)\frac{d}{dx}(3x^3+27)+\frac{d}{dx}\left(\frac{1}{x}+\frac{1}{x^2}\right)\cdot(3x^3+27)$$

$$=\left(\frac{1}{x}+\frac{1}{x^2}\right)(9x^2)+\left(\frac{-1}{x^2}-\frac{2}{x^3}\right)\cdot(3x^3+27)$$

$$=9x+9-3x-\frac{27}{x^2}-6-\frac{54}{x^3}$$

$$=6x+3-\frac{27}{x^2}-\frac{54}{x^3}$$

(26) Find $\left.\frac{dy}{dx}\right|_{x=1}$ $\quad y=\frac{4x+1}{x^2-5}$

$$\left.\frac{dy}{dx}\right|_{x=1}=\left.\frac{(x^2-5)\frac{d}{dx}(4x+1)-(4x+1)\frac{d}{dx}(x^2-5)}{(x^2-5)^2}\right|_{x=1}=\left.\frac{(x^2-5)\cdot(4)-(4x+1)\cdot(2x)}{(x^2-5)^2}\right|_{x=1}$$

$$=\left.\frac{4x^2-20-8x^2-2x}{(x^2-5)^2}\right|_{x=1}=\frac{4-20-8-2}{(1-5)^2}=\frac{-26}{16}=-\frac{13}{8}$$

(36) Find $g'(3)$ given that $f(3)=-2$ and $f'(3)=4$

(a) $g(x)=3x^2-5f(x)$

$g'(x)=6x-5f'(x)$

$g'(3)=6\cdot3-5(4)=18-20=-2$

(b) $g(x)=\frac{2x+1}{f(x)}$

$$g'(x)=\frac{f(x)\cdot\frac{d}{dx}(2x+1)-(2x+1)\cdot f'(x)}{[f(x)]^2}=\frac{f(x)\cdot2-(2x+1)f'(x)}{[f(x)]^2}$$

$$g'(3)=\frac{(-2)\cdot2-(2\cdot3+1)(4)}{(-2)^2}=\frac{-4-28}{4}=-8$$

(38) Find an equation for the line that is tangent to $y=\frac{1-x}{1+x}$ at $x=2$

$$\left.\frac{dy}{dx}\right|_{x=2} = \left.\frac{(1+x)\frac{d}{dx}(1-x)-(1-x)\frac{d}{dx}(1+x)}{(1+x)^2}\right|_{x=2} = \left.\frac{-1-x-1+x}{(1+x)^2}\right|_{x=2} = \left.\frac{-2}{(1+x)^2}\right|_{x=2} = \frac{-2}{9}$$

$$\left.y\right|_{x=2} = \frac{1-2}{1+2} = \frac{-1}{3}$$

$$m=\frac{y-y_0}{x-x_0} \Rightarrow -\frac{2}{9}=\frac{y-(-\frac{1}{3})}{x-2} \Rightarrow -2x+4=9y+3$$

Eqn: $2x+9y=1$ or $y=-\frac{2}{9}x+\frac{1}{9}$

(42) Find $\frac{d^2y}{dx^2}$

(a) $y=4x^7-5x^3+2x$

$$\frac{dy}{dx}=28x^3-15x^2+2$$

$$\frac{d^2y}{dx^2}=84x^2-30x$$

(b) $y=3x+2$

$$\frac{dy}{dx}=3$$

$$\frac{d^2y}{dx^2}=0$$

(c) $y=\frac{3x-2}{5x}$ for $x\neq 0$

$$\frac{dy}{dx}=\frac{5x\cdot\frac{d}{dx}(3x-2)-(3x-2)\frac{d}{dx}(5x)}{(5x)^2}$$

$$=\frac{15x-15x+10}{25x^2}=\frac{10}{25x^2}$$

$$\frac{d^2y}{dx^2}=\frac{25x^2\cdot\frac{d}{dx}(10)-10\frac{d}{dx}(25x^2)}{(25x^2)^2}$$

$$=\frac{-500x}{625x^4}=\frac{-4}{5x^3}$$

(d) $y=(x^3-5)(2x+3)$

$$\frac{dy}{dx}=(x^3-5)\frac{d}{dx}(2x+3)+(2x+3)\frac{d}{dx}(x^3-5)$$

$$=2x^3-10+6x^3+9x^2=8x^3+9x^2-10$$

$$\frac{d^2y}{dx^2}=24x^2+9x$$

(48) Show that if $x\neq 0$ and $y=\frac{1}{x}$, then $x^3y''+x^2y'-xy=0$

$$y=\frac{1}{x}=x^{-1}$$

$$y'=-x^{-2}=\frac{-1}{x^2}$$

$$y''=2x^{-3}=\frac{2}{x^3}$$

$$x^3\left(\frac{2}{x^3}\right)+x^2\left(\frac{-1}{x^2}\right)-x\left(\frac{1}{x}\right)$$

$$\Downarrow$$

$$2-1-1$$

$$\Downarrow$$

$$0 \checkmark$$

(58) Find **k** if $y=x^2+k$ is tangent to $y=2x$

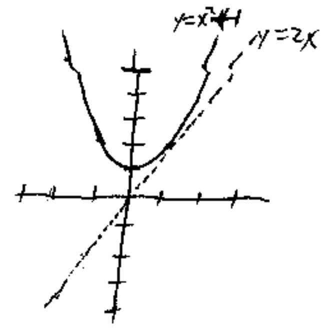
$\underbrace{\frac{dy}{dx} \text{ must be } 2}$ and $\underbrace{\text{the curves must intersect}}$ (at one point)

$\frac{dy}{dx} = 2x = 2$ $\qquad$ $x^2+k=2x$

$\therefore x=1$ $\qquad$ $k=2x-x^2$

$k=2(1)-1=1$

$\therefore k=1$ $\xrightarrow[\text{graph}]{\text{check}}$



(71) Apply the product rule (3.3.5) twice to show that if f, g and h are differentiable functions, then $f \cdot g \cdot h$ is differentiable and $(f\cdot g\cdot h)' = f'\cdot g\cdot h + f\cdot g'\cdot h + f\cdot g\cdot h'$

$$(f\cdot g\cdot h)' = [(f\cdot g)\cdot h]' = (f\cdot g)'\cdot h + (f\cdot g)h' = h\cdot[(f\cdot g)'] + f\cdot g\cdot h'$$

$$= h\cdot[f'\cdot g + g'\cdot f] + f\cdot g\cdot h' = f'\cdot g\cdot h + f\cdot g'h + f\cdot g\cdot h' \;//$$