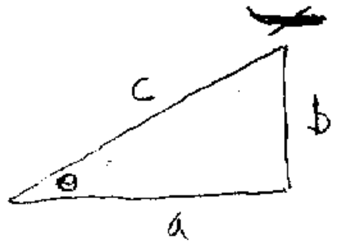
# Solution Set 14

## Section 4.6

**20**




Given: $b = 4000$ ft
$\theta = 30^\circ$
$\frac{da}{dt} = 300$ mi/h

(a) The first step is to convert $\theta$ to rad and $\frac{da}{dt}$ to feet/second

$\theta = \frac{\pi}{6}$ rad $\quad \frac{da}{dt} = \frac{300 \text{ mi}}{h} \cdot \frac{h}{3600 \text{ s}} \cdot \frac{5280 \text{ ft}}{\text{mi}} = 440$ ft/s

Next, using trig and the fact that $\theta = \frac{\pi}{6}$ and $b = 4000$ ft
we find that $a = 4000\sqrt{3}$ ft and $c = 8000$ ft

We know that ~~sin~~ $\tan\theta = \frac{4000}{a}$

Differentiating implicitly gives $\sec^2\theta \, \frac{d\theta}{dt} = \frac{-4000}{a^2}\frac{da}{dt}$

$$\frac{d\theta}{dt} = \frac{-4000 \cdot \frac{da}{dt}}{\sec^2\theta \; a^2} = \frac{-4000 \cdot 440}{(4000\sqrt{3})^2 \cdot \frac{4}{3}} = -0.0275 \text{ rad/s}$$

Converting to degrees gives $\theta = -1.58$ deg/s

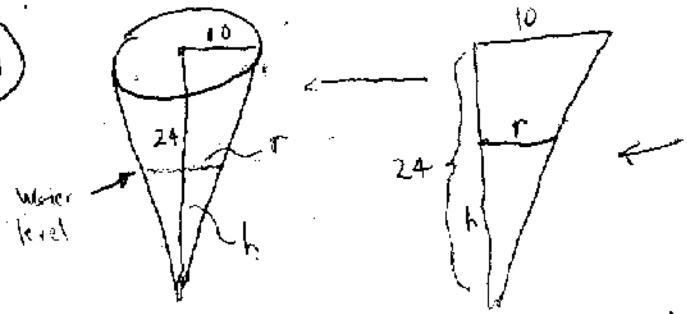
(b) We know that $\sin\theta = \frac{4000}{c}$. Differentiating gives $\cos\theta \frac{d\theta}{dt} = \frac{-4000}{c^2}\frac{dc}{dt}$

$$\frac{dc}{dt} = \frac{\cos\theta \, \frac{d\theta}{dt} \, c^2}{-4000} = \frac{\frac{\sqrt{3}}{2} \cdot (-0.0275) \cdot (8000)^2}{-4000} = 381 \text{ ft/s}$$

Alternatively, you could differentiate $a^2 + b^2 = c^2$ or $a^2 + 4000^2 = c^2$
to get $2a\frac{da}{dt} = 2c\frac{dc}{dt}$ so $\frac{dc}{dt} = \frac{2a\frac{da}{dt}}{2c} = \frac{2(4000\sqrt{3})(440)}{2 \cdot 8000} = 381$ ft/s

Solving:

(21)




Note that $\frac{r}{h} = \frac{10}{24}$ because these two triangles are similar

We are given $h = 16$ ft and $\frac{dV}{dt} = 20 \frac{ft^3}{min}$

We can substitute $\frac{r}{h} = \frac{10}{24}$ or $r = \frac{5}{12}h$ into $V = \frac{\pi}{3} r^2 h$

to give $V = \frac{\pi}{3}\left(\frac{25}{144}\right) h^3$ therefore $\frac{dV}{dt} = \frac{\pi}{3}\left(\frac{25}{144}\right) 3h^2 \frac{dh}{dt}$

so $\frac{dh}{dt} = \frac{\frac{dv}{dt} \cdot 3 \cdot 144}{\pi \cdot 25 \cdot 3 \cdot h^2}$ where $\frac{dv}{dt} = 20$ and $h = 16$

$\frac{dh}{dt} = \frac{9}{20\pi}$

(37) We want to find all $(x,y)$ on the curve $y = x\ln x$ which satisfy $3\frac{dx}{dt} = \frac{dy}{dt}$ Multiplying by $\frac{dt}{dx}$ gives

$3\frac{dx}{dt}\frac{dt}{dx} = \frac{dy}{dt}\frac{dt}{dx}$ so $3\frac{dx}{dx} = \frac{dy}{dx}$ and $3 = \frac{dy}{dx}$

Differentiating $y = x\ln x$ gives $\frac{dy}{dx} = \ln x + \frac{x}{x} = \ln x + 1$

Substituting $\frac{dy}{dx} = 3$ gives $3 = \ln x + 1$ $\quad 2 = \ln x \quad x = e^2$

Alternatively, if you did not notice that $\frac{dy}{dx} = 3$ you could differentiate $y = x\ln x$ with respect to $t$

$\frac{dy}{dt} = \frac{dx}{dt}\ln x + x\left(\frac{1}{x}\right)\frac{dx}{dt}$ $\quad \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \ln x + 1$

$3 = \ln x + 1 \quad 2 = \ln x \quad x = e^2$

(41) $V = \frac{4}{3}\pi r^3$

The meteorite burns at a ~~constant~~ rate that is proportional to its surface area. The rate at which it burns is $\frac{dV}{dt}$ and ~~the~~ surface area is $4\pi r^2$ so we know that

$\frac{dV}{dt} = 4\pi r^2 k$ where $k$ is some constant of proportionality.

By differentiating $V = \frac{4}{3}\pi r^3$ with respect to time we get

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$ Now we know that $4\pi r^2 k = 4\pi r^2 \frac{dr}{dt}$

So this means that $k = \frac{dr}{dt}$ and radius decreases at a constant rate because $\frac{dr}{dt}$ is constant. ~~and~~ ~~As~~