# Solution Set 23

## Section 7.2

(23) $\int \sec x(\sec x + \tan x)\,dx = \int(\sec^2 x + \sec x \tan x)\,dx = \int \sec^2 x\,dx + \int \sec x \tan x\,dx$

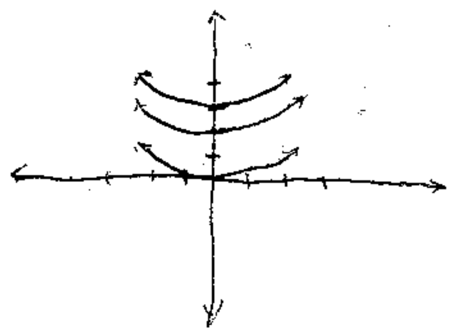
$= \tan x + \sec x$

Check: $\frac{d}{dx}(\tan x + \sec x) = \sec^2 x + \sec x \tan x = \sec x(\sec x + \tan x)$

(33) $f(x) = x$

$F(x) = \frac{1}{2}x^2 + C$ where $C$ is a constant

(a)



(b) If the curve passes through $(4,7)$, this means that $F(4) = 7$

$F(4) = \frac{1}{2}(4^2) + C = 7$

$8 + C = 7$

$C = -1$

$F(x) = \frac{1}{2}x^2 - 1$

(38) If the slope of the tangent line of $f(x)$ is $(x+1)^2$ then $f(x)$ must be the antiderivative of $(x+1)^2$

Note that $(x+1)^2$ is the same as $x^2 + 2x + 1$

$f(x) = \frac{1}{3}x^3 + x^2 + x + C$     check: $f'(x) = x^2 + 2x + 1$

$C$ is a constant and $f$ passes through $(-2, 8)$ so

$f(-2) = 8$

$8 = \frac{1}{3}(-2^3) + (-2^2) - 2 + C$

$8 = -\frac{8}{3} + 2 + C$

$8 = -\frac{2}{3} + C$

$C = 8\frac{2}{3}$

$f(x) = \frac{1}{3}x^3 + x^2 + x + 8\frac{2}{3}$

(40) $\frac{dy}{dx} = \frac{1}{(2x)^3} = \frac{1}{8x^3} = \frac{1}{8}x^{-3}$

$y = \frac{-1}{16}x^{-2} + C$

$y(1)=0$ so $0 = \frac{-1}{16}(1^{-2}) + C$

$y = \frac{-1}{16}x^{-2} + \frac{1}{16}$ $C = \frac{1}{16}$

(42) $f''(x) = x + \cos x$

$\int f''(x)\,dx = \int (x + \cos x)\,dx$

$f'(x) = \frac{1}{2}x^2 + \sin x + C$

we know that $f'(0)=2$ so $2 = \frac{1}{2}(0^2) + \sin(0) + C$ $C = 2$

$f'(x) = \frac{1}{2}x^2 + \sin x + 2$

$\int f'(x)\,dx = \int \left(\frac{1}{2}x^2 + \sin x + 2\right) dx$

$f(x) = \frac{1}{6}x^3 - \cos x + 2x + C$

$f(0)=1 \Rightarrow 1 = \frac{1}{6}(0^3) - \cos 0 + 2\cdot 0 + C$

$1 = 0 - 1 + 0 + C$

$C = 2$

$f(x) = \frac{1}{6}x^3 - \cos x + 2x + 2$

## Section 7.3

(2) (a) $u = 4x+1$
$du = 4\,dx$

$\int \sec^2(4x+1)\,dx = \frac{1}{4}\int \sec^2(4x+1)\,4\,dx = \frac{1}{4}\int \sec^2 u\,du = \frac{1}{4}\tan u + C$

$= \frac{1}{4}\tan(4x+1) + C$

(b) $u = 1+2y^2$
$du = 4y\,dy$

$\int y\sqrt{1+2y^2}\,dy = \frac{1}{4}\int \sqrt{1+2y^2}\,(4y\,dy) = \frac{1}{4}\int \sqrt{u}\,du = \frac{1}{4}\left(\frac{2}{3}u^{\frac{3}{2}}\right) + C$

$= \frac{2}{12}u^{\frac{3}{2}} + C = \frac{1}{6}u^{\frac{3}{2}} + C = \frac{1}{6}(1+2y^2)^{\frac{3}{2}} + C$

(c) $u = \sin \pi\theta$
$du = \pi \cos \pi\theta \, d\theta$

$$\int \sqrt{\sin \pi\theta} \cos \pi\theta \, d\theta = \frac{1}{\pi} \int \sqrt{\sin \pi\theta} \, \pi \cos \pi\theta \, d\theta$$
$$= \frac{1}{\pi} \int \sqrt{u} \, du = \frac{1}{\pi}\left(\frac{2}{3} u^{3/2}\right) + C = \frac{2}{3\pi} u^{3/2} + C$$
$$= \frac{2}{3\pi} (\sin \pi\theta)^{3/2} + C$$

(d) $u = x^2 + 7x + 3$
$du = (2x+7) \, dx$

$$\int (2x+7)(x^2+7+3)^{4/5} \, dx = \int u^{4/5} \, du = \frac{5}{9} u^{9/5} + C$$
$$= \frac{5}{9} (x^2 + 7x + 3)^{5/9} + C$$

(e) $u = 1 + e^x$
$du = e^x \, dx$

$$\int \frac{e^x \, dx}{1+e^x} = \int \frac{1}{u} \, du = \ln u + C = \ln(1+e^x) + C$$

5 $\int e^{2x} \, dx$

$u = 2x$
$du = 2 \, d\cancel{x}$

$$\int e^{2x} \, dx = \frac{1}{2} \int e^{2x} \, 2dx = \frac{1}{2} \int e^u \, du = \frac{1}{2} e^u + C$$
$$= \frac{1}{2} e^{2x} + C$$

8 $u = 3x - 1$
$du = 3 \, dx$

$$\int (3x-1)^5 \, dx = \frac{1}{3} \int (3x-1)^5 \, 3dx = \frac{1}{3} \int u^5 \, du = \frac{1}{3}\left(\frac{1}{6} u^6\right) + C$$
$$= \frac{1}{18} (3x-1)^6 + C$$

10 $u = 3x$
$du = 3 \, dx$

$$\int \sin 3x \, dx = \frac{1}{3} \int (\sin 3x) \, 3dx = \frac{1}{3} \int \sin u \, du = \frac{1}{3}(-\cos u) + C$$
$$= -\frac{1}{3} \cos 3x + C$$