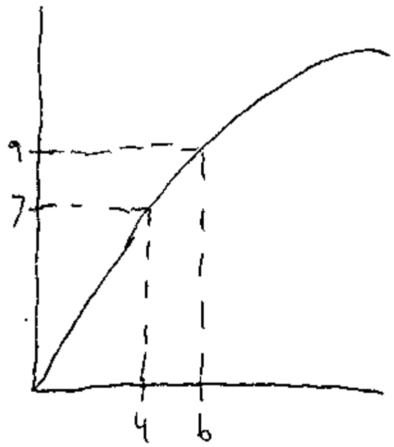
HW #28 Solutions

$\boxed{7.7}$ 5)



$$V(t) = V_0 + \underbrace{\int_0^t a(t')\,dt'}_{\text{Estimate area under the curve by counting boxes to evaluate}}$$

(a) $V(4) \approx 20 + 15 = 35\ m/s$

(b) $V(6) \approx 20 + 31 = 51\ m/s$

6) The particle is speeding up at both times because its velocity is positive and its acceleration is positive.

9) (a) $v(t) = s'(t) = 2t - 3$

$s(t) = \int v(t)\,dt = t^2 - 3t + c$

$s(1) = (1)^2 - 3(1) + c = 5 \longrightarrow c = 7$

$s(t) = t^2 - 3t + 7$

(b) $a(t) = \cos t$

$v(t) = \int a(t)\,dt = \sin t + c$

$v(\pi/2) = \sin(\pi/2) + c = 2 \longrightarrow c = 1$

$v(t) = \sin t + 1$

$s(t) = \int v(t)\,dt = -\cos t + t + c$

$s(\pi/2) = -\cos \pi/2 + \frac{\pi}{2} = 0 \longrightarrow c = -\frac{\pi}{2}$

$s(t) = -\cos t + t - \frac{\pi}{2}$

HW # 28 continued

16) $a(t) = t - 2$

$v(t) = v_0 + \int a(t)\,dt = \frac{1}{2}t^2 - 2t$

$s(t) = s_0 + \int v(t)\,dt = \frac{1}{6}t^3 - t^2$

To find distance traveled, first find when $v(t) = 0$

$\frac{1}{2}t^2 - 2t = 0 \longrightarrow \frac{1}{2}t\,(t-4) = 0 \longrightarrow t = 0, 4$

From $t=1$ to $t=4$, particle travels $|s(1) - s(4)|$ meters

$|s(1) - s(4)| = \left|\left(\frac{1}{6} - 1\right) - \left(\frac{64}{6} - 16\right)\right| = \frac{27}{6}$ m

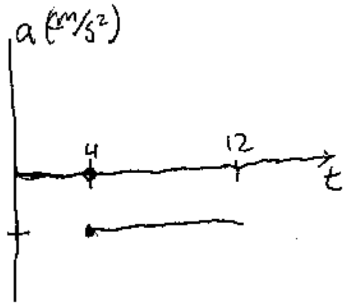
From $t = 4$ to $t = 5$

$|s(4) - s(5)| = \left|\left(\frac{64}{6} - 16\right) - \left(\frac{125}{6} - 25\right)\right| = \frac{7}{6}$ m

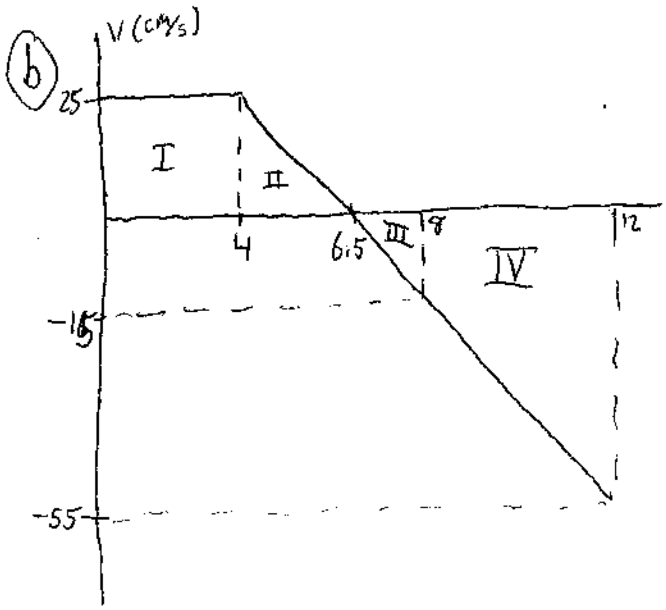
Total distance $= \frac{27}{6} + \frac{7}{6} = \boxed{17/3 \text{ m}}$

Displacement $= s(5) - s(1) = -\frac{25}{6} - \left(-\frac{5}{6}\right) = \boxed{-\frac{10}{3}\text{ m}}$

29) (a)



(b)



(c) $s(t) = \int v(t)\,dt$ Think in terms of area

$s(8) = \text{I} + \text{II} + (-\text{III})$

$= 25(4) + \frac{1}{2}(2.5)(25) - \frac{1}{2}(1.5)(15)$

$= \boxed{120 \text{ cm}}$

$s(12) = \text{I} + \text{II} - (\text{III} + \text{IV})$

$= 25(4) + \frac{1}{2}(2.5)(25) - \frac{1}{2}(5.5)(55)$

$= \boxed{-20 \text{ cm}}$

(d) Max displacement when $v = 0$

$s(6.5) = 25(4) + \frac{1}{2}(2.5)(25)$

$= \boxed{131.25 \text{ cm}}$

HW #28 continued

36) $a(t) = \begin{cases} 4, & 0 \le t \le 2 \\ 0, & t > 2 \end{cases}$

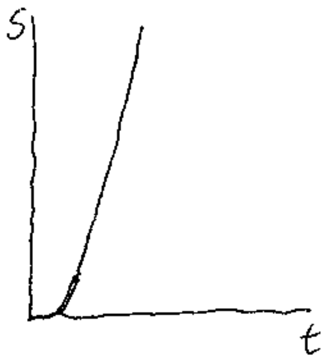
$V(t) = \begin{cases} 4t, & 0 \le t \le 2 \\ 8, & t > 2 \end{cases}$

When $0 \le t \le 2$, $s(t) = \int v(t)dt = 2t^2$

When $t > 2$, $s(t) = s(2) + 8(t-2) = 8t - 8$

(a) $s(t) = 100 \Rightarrow 8t - 8 = 100 \quad t = \frac{108}{8} = \boxed{\frac{27}{2} \text{ s}}$

(b)



37)

<u>car</u>	<u>truck</u>
$a_1(t) = 2$	$a_2(t) = 0$
$V_1(t) = 2t$	$V_2(t) = 50$
$S_1(t) = t^2$	$S_2(t) = 50t + 5000$

$S_1(t) = S_2(t)$

$t^2 = 50t + 5000$

$t^2 - 50t - 5000 = 0$

$(t - 100)(t + 50) = 0$

$t = \boxed{100 \text{ s}}, \cancel{-50}$

$S(100) = \boxed{10{,}000 \text{ ft}}$

50) $\frac{1}{2-(-1)} \int_{-1}^{2} x^2 dx = \frac{1}{3}\left(\frac{1}{3}x^3 \Big|_{-1}^{2}\right) = \frac{1}{9}(8 - (-1)) = \boxed{1}$

51) $\frac{1}{\pi - 0} \int_0^{\pi} \sin x dx = \frac{1}{\pi}\left(-\cos x \Big|_0^{\pi}\right) = \frac{1}{\pi}(-\cos \pi + \cos 0) = \boxed{\frac{2}{\pi}}$