Math 1a - Exam 1 Review Problems
Spring 2000

1) Find the following derivatives:

a) $g(t) = \tan t\,(2+t^3)^5$

b) $f(x) = \tan\sqrt{\dfrac{x-1}{x+1}}$

c) $f(x) = \cos^2(\frac{3}{x})$

d) Find $\dfrac{dy}{dx}$ if $y = \cos^4(x^5)$

e) Find $f'(x)$ if $f(x) = \sin^2 3x + \cos^2 3x$

f) Find $f'(1)$ for $f(x) = \dfrac{3}{\sqrt[3]{x}} - x - \dfrac{1}{x}$

g) Find $\dfrac{dy}{dx}$ for the relation $x^2 + xy + 2y = 1$

2) Suppose that all we know about a continuous, differentiable function $f(x)$ is the following numerical data:

x	0.2	0.4	0.6	0.8	1.0
$f(x)$	20	25	27	21	15

Give estimates for $f'(0.6)$ and $f''(0.6)$.

3) Consider the function

$$f(x) = \begin{cases} x^2 \sin(\frac{1}{x}) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}.$$

a) What is $f'(x)$ for $x \neq 0$?
b) What is $f'(0)$?

4) Which of the following expressions represent the derivative of f at $x = a$?

a) $\displaystyle\lim_{h\to 0}\frac{f(a+2h)-f(a)}{2h}$

b) $\displaystyle\lim_{x\to a}\frac{f(x)}{x}$

c) $\displaystyle\lim_{h\to 0}\frac{f(a+h)-f(a-h)}{2h}$

d) $\displaystyle\lim_{x\to a}\frac{f(x)-f(a)}{x-a}$

e) $\displaystyle\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$

5) (a) Use the definition of the derivative to compute $f'(0)$ for $f(x) = \dfrac{1}{2x+1}$.

(b) Use the definition of the derivative to show that if $f(x) = \dfrac{1}{x^2}$, then $f'(x) = -\dfrac{2}{x^3}$.

6) Find the equations for two lines through the origin that are tangent to the curve with equation: $x^2 - 4x + y^2 + 3 = 0$.

7) Consider the parabola $y = x^2$. At what point on this parabola is the tangent line parallel to the line connecting the points (1,1) and (3,9) on this parabola?

8) An object travels on a straight line. Its position on that line is given by $s(t) = \frac{2}{3}t^3 - 7t^2 + 20t + 8$, starting at $t = 0$. When its velocity is positive, it is moving to the right. When its velocity is negative, it is moving to the left.
a) Find a formula for its velocity at time t.
b) Find a formula for its acceleration at time t.
c) When is it moving to the right?
d) When is its velocity decreasing?
e) When does the object change direction?

9) Suppose that $f(x)$ is a differentiable function whose derivative satisfies $f'(x) = (x^2-1)^{-1/2}$.
Find $\dfrac{d}{dx}\left[f(\sec(x))\right]$ for $x \in (0, \frac{\pi}{2})$.

10) Find $\dfrac{dy}{dx}$ at the point (0,-1) for the relation $\sec(y^2 + y) = xy + 1$.

11) a) Use the idea of linear approximation for the function $f(x) = \sqrt{x}$ and fact that $(2.5)^2 = 6.25$ to approximate $\sqrt{6}$.
b) Is this approximation less than or greater than the actual value of $\sqrt{6}$? Why?

Math 1a Exam #1: Monday, October 25, 1999

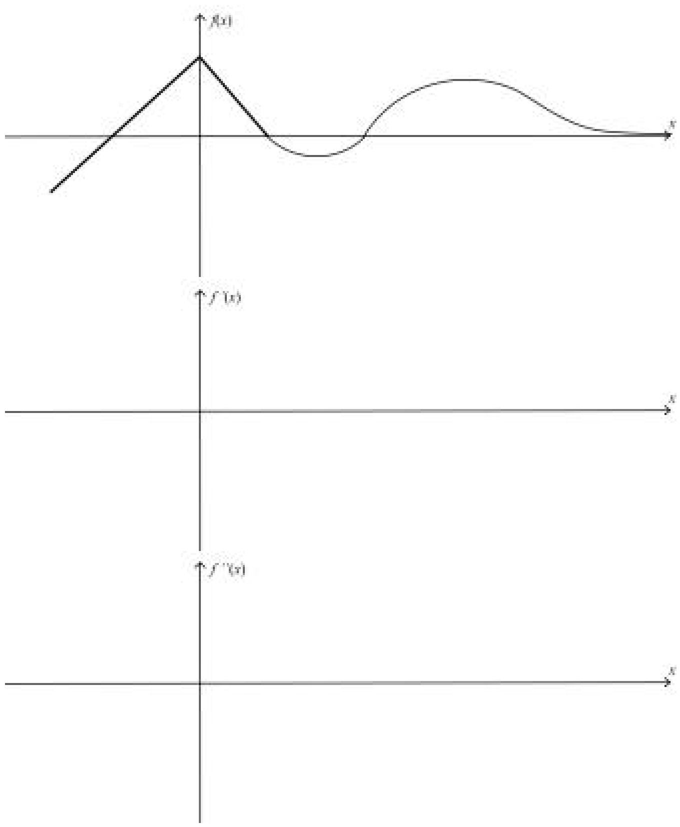
(1) Find the following limits, if they exist:

(a) $\lim_{x\to 2} \dfrac{x^2+3x}{x^3+5x+1}$

(b) $\lim_{t\to -3} \dfrac{|t|}{|4t+6|}$

(c) $\lim_{w\to -1} \dfrac{3+4w+w^2}{1+w}$

2) Given the graph of the function $y = f(x)$ as shown, graph its first and second derivatives on the given axes, marking all noteworthy points appropriately.



(3) Let $h(x) = \begin{cases} 3x-2, & x \le 0 \\ x^2-2, & x > 0 \end{cases}$

(a) Sketch the graph of h.

(b) Find $\lim_{x\to 0} h(x)$ if it exists.

(c) Is h continuous? Why or why not?

(d) Is h differentiable at $x = 0$? Why or why not?

(4) Let $f(x) = x^2 + 2x$.

a) What is the domain of f?

b) Find the equation of the tangent line at $x = 2$ using the definition of the derivative.

(5) Find derivatives of the following functions using any method.

(a) $f(x) = \dfrac{1}{\sqrt{x}}((\sqrt{x})^5 + 1)$

(b) $f(x) = x^3 \cos 5x$

(c) $f(x) = \dfrac{\sin x}{3x+2}$

(6) Find the equation of the tangent line to the curve $x^2y + 2y^3 = 3x + 2y + 54$ at the point where $(x, y) = (2,3)$.

(7) A particle moves with displacement

$s(t) = 16(1 - \dfrac{1}{t+1})$ meters.

(a) Find the average velocity from $t = 1$ to $t = 3$.

(b) Find the instantaneous velocity at $t = 3$.

8) Given that the side length of a cube is known to be 8 centimeters with a possible error of ± 2%, use linear approximation to estimate the percentage error in the surface area of the cube.

Note: You should not draw too many conclusions from this old exam about what may or may not be on this semester's exam or what the difficulty level of the exam problems will be. Consult your homework and class notes.