Solution Set 8

(10) (a) $y = x^{-1}$ $\quad dy = -x^{-2}\,dx = \dfrac{-dx}{x^2}$

(b) $y = 5\tan x$ $\quad dy = 5\sec^2 x\,dx$

(13) (a) Formula 6 : $f(x) \approx f(x_0) + f'(x_0)(x-x_0)$

In this case $f(x) = x^3$ and $x_0 = 1$, therefore $f(x_0) = 1$ and $f'(x) = 3x^2$ so $f'(x_0) = f'(1) = 3$

Substituting into eq 6, we have

$$f(x) \approx 1 + 3(x-1)$$

(b) Formula 7: $f(x_0 + \Delta x) \approx f(x_0) + f'(x_0)\Delta x$

using $f(x_0) = 1$ and $f'(x_0) = 3$ we get

$$f(1+\Delta x) \approx 1 + 3\Delta x$$

(c) From part (a) we have

$$(1.02)^3 = f(1.02) \approx 1 + 3(1.02-1) = 1.06$$

From part (b) we have

$$(1+0.02)^3 = f(1+0.02) \approx 1 + 3(0.02) = 1.06$$

(19) Formula 6: $f(x) \approx f(x_0) + f'(x_0)(x-x_0)$

In this case $f(x) = \tan x$ $\quad f(0) = 0$ $\quad f'(x) = \sec^2 x$ $\quad f'(0) = \sec^2(0) = \dfrac{1}{\cos^2(0)} = 1$

substituting:

$$\tan x \approx 0 + 1\cdot(x-0)$$

$$\tan x \approx x$$

(20) In this problem $f(x) = (x+1)^{-1}$ $\quad f(0) = 1$ $\quad f'(x) = -(x+1)^{-2}$ and $f'(0) = -1$

Substituting into formula 6, we get

$$\frac{1}{x+1} \approx 1 + (-1)(x-0)$$

$$\frac{1}{x+1} \approx 1 - x$$

(26) (a) As shown in problem 19, the linear approximation to $\tan x$ at $x_0 = 0$ is

$$\tan x \approx x$$

Note that 2° is equal to $\pi/90$ radians.

$$\tan \tfrac{\pi}{90} \approx \tfrac{\pi}{90} \approx 0.03491$$

Using a calculator, we find that $\tan \frac{\pi}{90} \approx 0.03492$

(b) To approximate $\tan 61^\circ$ choose $x_0 = 60^\circ = \frac{\pi}{3}$ radians

(c) Once again, formula 6 is as follows $f(x) \approx f(x_0) + f'(x_0)(x - x_0)$

In this problem $f(x) = \tan(x)$ $f(x_0) = \tan(60^\circ) = \sqrt{3}$

$f'(x) = \sec^2(x)$ so $f'(x_0) = \sec^2(60^\circ) = 4$

Substituting:

$$\tan(x) \approx \sqrt{3} + 4\left(x - \tfrac{\pi}{3}\right)$$

Since $61^\circ = \frac{61\pi}{180}$ radians, we have

$$\tan(61^\circ) \approx \sqrt{3} + 4\left(\tfrac{61\pi}{180} - \tfrac{\pi}{3}\right)$$

$$\tan(61^\circ) \approx \sqrt{3} + \tfrac{\pi}{45}$$

$$\tan(61^\circ) \approx 1.802$$

A calculator gives the value $\tan(61^\circ) \approx 1.804$

(36) (a) We are trying to approximate $(1+x)^k$ so consider the function $f(x) = x^k$

Choose $x_0 = 1$ and apply formula 7

$$f(x_0 + \Delta x) \approx f(x_0) + f'(x_0)\Delta x \qquad \text{where } x_0 = 1 \quad f(x_0) = 1^k = 1$$
$$f'(x) = kx^{k-1} \quad f'(x_0) = k(1)^{k-1} = k$$

$$f(1 + \Delta x) \approx 1 + k\Delta x$$

$$(1+\Delta x)^k \approx 1 + k\Delta x$$

To approximate $(1.001)^{37}$ we put $\Delta x = 0.001$ and $k = 37$ into the formula above

$$(1.001)^{37} \approx 1 + (37)(0.001) = 1.037$$

(b) A calculator gives the value 1.03767

(c) The formula gives $(1.1)^{37} \approx 1 + (37)(0.1) = 4.7$ whereas a calculator gives $1.1^{37} = 34.0039$ The estimate is bad because it is a linear approximation of a function which has a rapidly changing gradient. 1.1 is too far from the point of tangency to yield a good result.

45 (a) $A = s^2$ so $dA = 2s\,ds$

Given that $S = 10\text{ft}$ with $ds = \pm 0.1\text{ ft}$, we know that the error in the area will be approximated by dA

$$dA = 2\cdot(10\text{ft})(\pm 0.1\text{ft}) = \pm 2\text{ft}^2$$

(b) Side : $\dfrac{\pm 0.1\text{ft}}{10\text{ ft}} = \pm 1.0\%$

Area : $\dfrac{\pm 2\text{ft}^2}{100\text{ ft}^2} = \pm 2.0\%$

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$V = s^3$

We are given that the side has a percentage error of $\pm 2\%$. So this means that $\dfrac{ds}{s} = \pm 2\%$

Using $V = s^3$, we find that $dV = 3s^2\,ds$

$$\frac{dV}{s^3} = \frac{3s^2 ds}{s^3}$$

$$\frac{dV}{V} = 3\frac{ds}{s}$$

$$\frac{dV}{V} = 3\cdot(\pm 2\%) = \pm 6\%$$