Section 4.3

3. $y = \left(\frac{x-1}{x+2}\right)^{3/2}$

let $u = \frac{x-1}{x+2}$.

Then, by the quotient rule, $\frac{du}{dx} = \frac{(x+2)\cdot 1 - (x-1)\cdot 1}{(x+2)^2}$

$$= \frac{3}{(x+2)^2} .$$

By the chain rule, $\frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx} = \frac{3}{2}u^{\frac{1}{2}}\cdot\frac{3}{(x+2)^2}$

$$= \boxed{\frac{3}{2}\left(\frac{x-1}{x+2}\right)^{\frac{1}{2}}\cdot\frac{3}{(x+2)^2}}$$

9. $x^3 + xy - 2x = 1$.

(a) $\frac{d}{dx}(x^3 + xy - 2x) = \frac{d}{dx}(1) = 0$

$\Rightarrow 3x^2 + x\cdot\frac{dy}{dx} + y - 2 = 0$

$\Rightarrow x\cdot\frac{dy}{dx} = 2 - y - 3x^2$

$\Rightarrow \boxed{\frac{dy}{dx} = \frac{2-y-3x^2}{x}}$

(b) $xy = 1 + 2x - x^3$

$\Rightarrow y = \frac{1+2x-x^3}{x}$

By the quotient rule, $\frac{dy}{dx} = \frac{x(2-3x^2) - (1+2x-x^3)\cdot 1}{x^2}$

Thus, $\frac{dy}{dx} = \boxed{\frac{-2x^3 - 1}{x^2}}$

(c) $\frac{dy}{dx} = \frac{2 - y - 3x^2}{x}$, $y = \frac{1+2x-x^3}{x}$

$\Rightarrow \frac{dy}{dx} = \frac{2 - \left(\frac{1+2x-x^3}{x}\right) - 3x^2}{x} \cdot \frac{x}{x}$

$= \frac{2x - (1+2x-x^3) - 3x^3}{x^2}$

$= \frac{-2x^3 - 1}{x^2}$

So it checks out.

12. $x^3 - y^3 = 6xy$

$\Rightarrow \frac{d}{dx}\left(x^3 - y^3\right) = \frac{d}{dx}(6xy)$

$\Rightarrow 3x^2 - 3y^2 \cdot \frac{dy}{dx} = \frac{d}{dx}(6xy)$ by the chain rule

$\Rightarrow 3x^2 - 3y^2 \cdot \frac{dy}{dx} = 6y + 6x \cdot \frac{dy}{dx}$ by the product rule

$\Rightarrow 3x^2 - 6y = 3y^2 \cdot \frac{dy}{dx} + 6x\frac{dy}{dx}$

$\Rightarrow \boxed{\frac{3x^2 - 6y}{3y^2 + 6x} = \frac{dy}{dx}}$

24. $2xy - y^2 = 3$

$\Rightarrow \frac{d}{dx}\left(2xy - y^2\right) = \frac{d}{dx}(3) = 0$

$\Rightarrow 2y + 2x \cdot \frac{dy}{dx} - 2y \cdot \frac{dy}{dx} = 0$ by the chain + product rules

$\Rightarrow 2y = 2y\frac{dy}{dx} - 2x\frac{dy}{dx}$

$\Rightarrow y = (y-x)\frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{y}{y-x}$

So, $\frac{d^2y}{dx^2} = \frac{\frac{dy}{dx}(y-x) - y\left(\frac{dy}{dx} - 1\right)}{(y-x)^2}$ by the quotient rule.

$= \frac{\frac{y}{y-x}(y-x) - y\left(\frac{y}{y-x} - 1\right)}{(y-x)^2}$ since $\frac{dy}{dx} = \frac{y}{y-x}$

$= \frac{y - y\left(\frac{y-(y-x)}{y-x}\right)}{(y-x)^2} = \frac{y(y-x) - yx}{(y-x)^3}$

Thus, $\boxed{\frac{d^2y}{dx^2} = \frac{y^2 - 2yx}{(y-x)^3}}$

40. The slope of the line $4x-3y+1=0$ is $\frac{4}{3}$.

If the tangent line to the curve is perpendicular to this line, it must have slope $-\frac{3}{4}$.

This happens when $\frac{dy}{dx} = -\frac{3}{4}$.

$$y^2 = 2x^3 \Rightarrow \frac{d}{dx}(y^2) = \frac{d}{dx}(2x^3)$$

$$\Rightarrow 2y \cdot \frac{dy}{dx} = 6x^2$$

$$\Rightarrow \frac{dy}{dx} = \frac{3x^2}{y}$$

So, we need (x,y) s.t. $y^2 = 2x^3$ and

$$-\frac{3}{4} = \frac{3x^2}{y} \Rightarrow -\frac{1}{4} = \frac{x^2}{y}$$

$$\Rightarrow \frac{1}{16} = \frac{x^4}{y^2}$$

$$\Rightarrow \frac{1}{16} = \frac{x^4}{2x^3}$$

$$\Rightarrow \frac{1}{8} = x.$$

$$-\frac{3}{4} = \frac{3x^2}{y} \Rightarrow -\frac{3}{4} = \frac{\frac{3}{64}}{y}$$

$$\Rightarrow y = \frac{\frac{3}{64}}{-\frac{3}{4}} = \frac{-1}{16}$$

This is on our curve, since $\left(\frac{-1}{16}\right)^2 = \frac{1}{32} = 2 \cdot \left(\frac{1}{8}\right)^3$.

Thus, our solution is $\boxed{\left(\frac{1}{8}, -\frac{1}{16}\right)}$

45. $2y^3t + t^3y = 1, \qquad \frac{dt}{dx} = \frac{1}{\cos t}.$

$$\frac{d}{dx}\left(2y^3 \cdot t + t^3y\right) = \frac{d}{dx}(1) = 0$$

$$\Rightarrow 2\left(3y^2 \frac{dy}{dx} \cdot t + y^3 \frac{dt}{dx}\right) + 3t^2 \cdot \frac{dt}{dx} y + t^3 \cdot \frac{dy}{dx} = 0.$$

$$\Rightarrow 6y^2t \frac{dy}{dx} + \frac{2y^3}{\cos t} + \frac{3t^2y}{\cos t} + t^3\frac{dy}{dx} = 0$$

$$\Rightarrow (6y^2t + t^3)\frac{dy}{dx} = \frac{-2y^3 - 3t^2y}{\cos t}$$

$$\Rightarrow \boxed{\frac{dy}{dx} = \frac{-2y^3 - 3t^2y}{\cos t\,(6y^2t + t^3)}}$$

We can't simplify it any further, since we don't know what t is.