Solution Set 15

Section 5.1

(47) (a) True

(b) False

Consider $f(x)=x$ and $g(x)=x$ these functions are increasing for all x but $f\cdot g = x^2$ and this function decreases for $x<0$

(54) $y = (1+x^2)^{-1}$ We are looking for the values of x for which the function is increasing and decreasing most rapidly. In other words we want to find the min and max of the function's derivative.

$$\frac{dy}{dx} = -(1+x^2)^{-2}(2x) = \frac{-2x}{(1+x^2)^2}$$

Now we look for critical points of $\frac{dy}{dx}$

$$\frac{d^2y}{dx^2} = \frac{(1+x^2)^2(-2) - (-2x)(2(1+x^2)(2x))}{(1+x^2)^4} = \frac{(1+x^2)(-2) - (-8x^2)}{(1+x^2)^3}$$

$$\frac{d^2y}{dx^2} = \frac{-2+6x^2}{(1+x^2)^3}$$ Critical points at $x = \pm\sqrt{1/3}$

To see whether these are relative maxima or minima, use the Second derivative test.

$$\frac{d^3y}{dx^3} = \frac{-24x(x^2-1)}{(1+x^2)^4}$$

at $x=-\sqrt{1/3}$ $\frac{d^3y}{dx^3} < 0$

at $x=\sqrt{1/3}$ $\frac{d^3y}{dx^3} > 0$

$x=-\sqrt{1/3}$ is a max and $x=\sqrt{1/3}$ is a min.

y increases most at $x=-\sqrt{1/3}$ and decreases most at $x=\sqrt{1/3}$

(56)