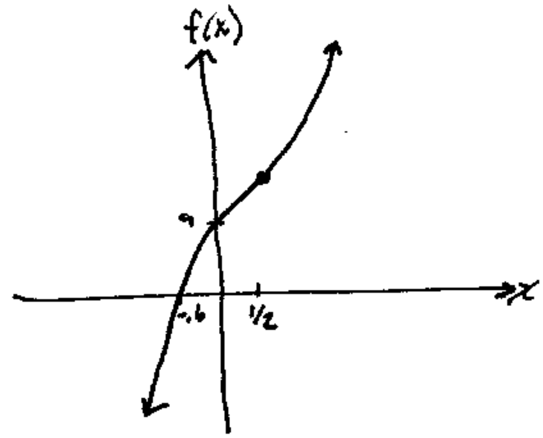
HW #16 §5.3 #4, 14, 19, 23, 40, 45, 47 Solutions
§4.7 #1, 10, 15, 27, 66

$\boxed{§5.3}$

#4) $f(x) = 2x^3 - 3x^2 + 12x + 9$

$f'(x) = 6x^2 - 6x + 12 = 6(x^2 - x + 2) \geq 0$ for all x

$f''(x) = 12x - 6 = 6(2x-1) = 0$ @ $x = 1/2$

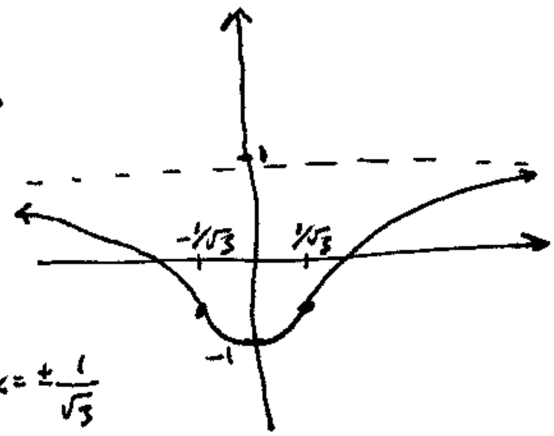


#14) $f(x) = \frac{x^2-1}{x^2+1}$

$$f'(x) = \frac{2x(x^2+1) - 2x(x^2-1)}{(x^2+1)^2} = \frac{4x}{(x^2+1)^2} = 0 \text{ @ } x = 0$$

$$f''(x) = \frac{4(x^2+1)^2 - 4x \cdot 2(x^2+1) \cdot 2x}{(x^2+1)^4}$$

$$= \frac{4(x^2+1)(x^2+1-4x^2)}{(x^2+1)^4} = \frac{4(1-3x^2)}{(x^2+1)^3} = 0 \text{ @ } x = \pm\frac{1}{\sqrt{3}}$$



$$\lim_{x\to\infty} f(x) = \lim_{x\to-\infty} f(x) = 1$$

#23) a) VI
b) I
c) III
d) V
e) IV
f) II

#40) $f(x) = xe^{-2x}$

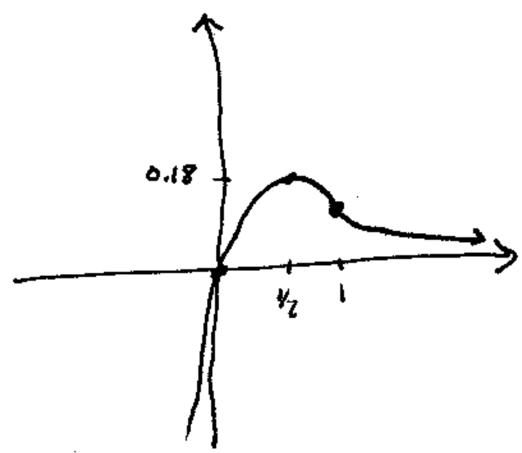
$f'(x) = e^{-2x} - 2xe^{-2x}$

$= e^{-2x}(1-2x) = 0$ @ $x = 1/2$

$f''(x) = -2e^{-2x} - 2e^{-2x}(1-2x)$

$= -2e^{-2x}(2-2x)$

$= -4e^{-2x}(1-x) = 0$ @ $x = 1$



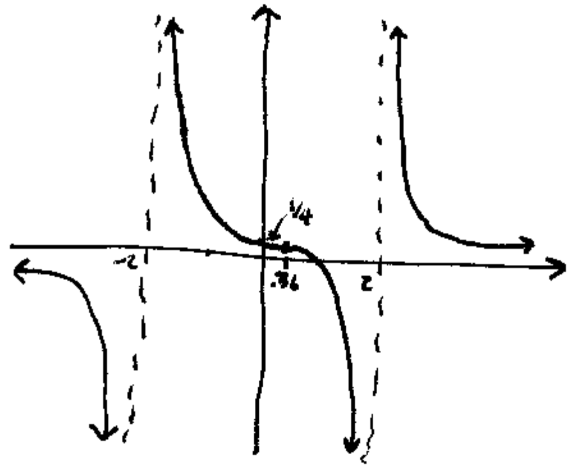
$$\lim_{x\to\infty} xe^{-2x} = \lim_{x\to\infty} \frac{x}{e^{2x}} = \lim_{x\to\infty} \frac{1}{2e^{2x}} = 0$$

$$\lim_{x\to-\infty} xe^{-2x} = -\infty$$

#19) $f(x) = \frac{x-1}{x^2-4} = 0$ @ $x=1$

undefined at $x = \pm 2$

$f'(x) = \frac{(x^2-4) - 2x(x-1)}{(x^2-4)^2} = \frac{-x^2+2x-4}{(x^2-4)^2}$ which never $= 0$.



$f''(x) = \frac{(x^2-4)^2(-2x+2) - (-x^2+2x-4)(2\cdot(x^2-4)\cdot 2x)}{(x^2-4)^4}$

$= \frac{(x^2-4)\left[(x^2-4)(-2x+2) - 4x(-x^2+2x-4)\right]}{(x^2-4)^4}$

$= \frac{2(x^3-3x^2+12x-4)}{(x^2-4)^3} = 0$ @ $x \approx 0.362$

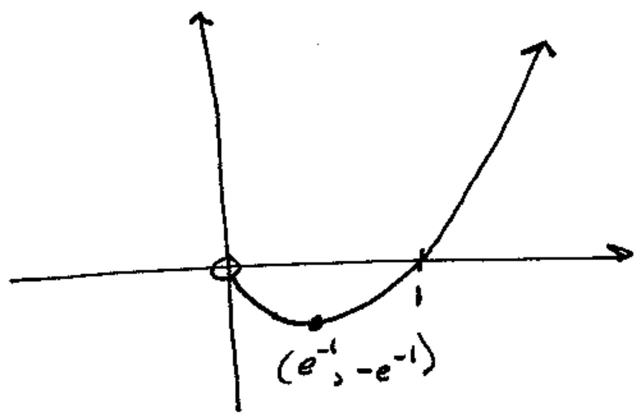
$\lim_{x\to\infty} f(x) = 0 = \lim_{x\to-\infty} f(x)$

#45) $f(x) = x \ln x$

$f'(x) = \ln x + x\cdot\frac{1}{x} = \ln x + 1$

$= 0$ @ $x = e^{-1}$

$f''(x) = 1/x \neq 0$ ever, always >0 for $x>0$



$\lim_{x\to\infty} f(x) = \infty$

$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \frac{\ln x}{1/x} = \lim_{x\to 0^+} \frac{1/x}{-1/x^2} = \lim_{x\to 0^+}(-x) = 0$

#47) $f(x) = \frac{\ln x}{x^2}$

$f'(x) = \frac{x - 2x\ln x}{x^4} = \frac{(1-2\ln x)}{x^3} = 0$ @ $x = e^{1/2}$

$f''(x) = \frac{-2x^2 - 3x^2(1-2\ln x)}{x^6}$

$= \frac{x^2(-2-3+6\ln x)}{x^6}$

$= \frac{6\ln x - 5}{x^4} = 0$ @ $x = {}^{5/6}$

$\lim_{x\to\infty} f(x) = 0$

$\lim_{x\to-\infty} f(x) = -\infty$