Math 1a HW #31

② $\int_1^4 \frac{1}{\sqrt{x}}\,dx$, $n=10$ $\quad \int_1^4 \frac{1}{\sqrt{x}}\,dx = \int_1^4 x^{-1/2}\,dx = \left.\frac{x^{1/2}}{1/2}\right|_1^4 = 2(\sqrt{4}-\sqrt{1}) = 2$ (exact value)

(a) Mid Point Approx $= \frac{4-1}{10}\left(f(1.15)+f(1.45)+\dots f(3.85)\right) = .3(.9325+.83045+\dots .509647)$

$\approx \underline{1.99837704849}$

Absolute error $= |2-1.99837704849| = \underline{0.00162295151}$

(b) Trapezoid Approx $\approx \frac{4-1}{2\cdot 10}\left(f(1)+2\cdot f(1.3)+\dots 2f(3.7)+f(4)\right) \approx \underline{2.0032609822}$

Absolute error $= |2-2.0032609822| = \underline{0.0032609822}$

(c) Simpson's rule $\approx \frac{4-1}{3\cdot 10}\left(f(1)+4f(1.3)+2f(1.6)+\dots +2f(3.4)+4f(3.7)+f(4)\right) \approx \underline{2.00007269759}$

Absolute error $= |2-2.00007269759| = \underline{0.00007269759}$

⑧ $f(x)=\frac{1}{\sqrt{x}}$ $\quad f'(x)=\frac{-x^{-3/2}}{2}$ $\quad f''(x)=\frac{3}{4}x^{-5/2}$ $\quad f'''(x)=\frac{-15}{8}x^{-7/2}$ $\quad f^{(4)}(x)=\frac{105}{16}x^{-9/2}$

since $\left|\frac{3}{4x^{5/2}}\right|$ is maximum when $x^{5/2}$ is minimum (which happens over $[1,4]$ when $x=1$), $K_2=\frac{3}{4}$

similarly $K_4=\frac{105}{16}$; $\Delta x=\frac{4-1}{10}=.3$; $b-a=4-1=3$ ~~because~~ $n=10$

(a) $|E_M| \le \frac{1}{24}K_2(b-a)(\Delta x)^2 = \frac{1}{24}\cdot\frac{3}{4}(3)(.3)^2 = \underline{.0084375}$

(b) $|E_T| \le \frac{1}{12}K_2(b-a)(\Delta x)^2 = 2|E_M| = \underline{.016875}$

(c) $|E_S| \le \frac{1}{180}K_4(b-a)(\Delta x)^4 = \frac{1}{180}\cdot\frac{105}{16}\cdot 3\cdot(.3)^4 = \underline{.0008859375}$

⑭ (b) $|E_T| \le 5\times 10^{-4}$ so $\frac{1}{12}\cdot\frac{3}{4}\cdot 3\cdot\left(\frac{3}{n}\right)^2 \le 5\times 10^{-4} \Leftrightarrow n^2 \ge 3375 \Leftrightarrow n \ge 58.0947$

so use $\underline{n}=59$.

(a) $|E_M| \le 5\times 10^{-4}$ so $\frac{1}{24}\left(\frac{3}{4}\right)(3)\left(\frac{3}{n}\right)^2 \le 5\times 10^{-4} \Leftrightarrow n^2 \ge 1687.5 \Leftrightarrow n \ge 41.07919$

so use $n=42$

(c) $|E_S| \le 5\times 10^{-4}$ so $\frac{1}{180}\left(\frac{105}{16}\right)(3)\left(\frac{3}{n}\right)^4 \le 5\times 10^{-4} \Leftrightarrow n^4 \ge 17718.75 \Leftrightarrow n \ge 11.5374$

so use $n=12$ (closest even number ≥ 11.5374)

(28) ~~(a)~~ Notice that when a curve is concave up over an interval, the trapezoid rule overestimates the true area (assuming the curve is always positive) (fig 1)

Similarly, notice that when a curve is concave down, the trapezoid rule underestimates the true area (assuming the curve is always positive) (fig 2)

fig 1

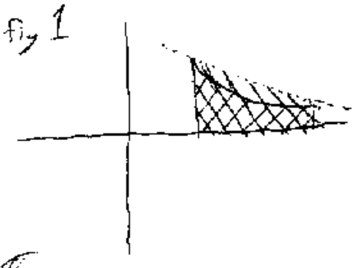
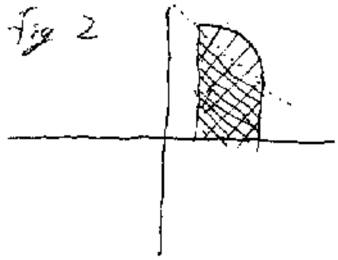


fig 2



~~(a)~~ $f(x) = e^{-x^2}$ $\quad f'(x) = -2xe^{-x^2}$ $\quad f''(x) = -2x(-2xe^{-x^2}) + e^{-x^2}(-2)$

$$= (4x^2 - 2)e^{-x^2}$$

Since $e^{-x^2} > 0$ for all x, the sign of $f''(x)$ (i.e. the concavity) is determined exclusively by the sign of $4x^2 - 2$. $4x^2 - 2 = 0$ when $x = \pm\sqrt{\frac{1}{2}} \approx \pm .707$, so $f''(x) \leq 0$ over the interval $[-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}]$

(a) $\int_1^2 e^{-x^2}dx$. Over $[1, 2]$, e^{-x^2} is concave down because $f''(x) > 0$. So the trapezoid rule's approximation will be less than the exact value of the integral

(b) $\int_0^{0.5} e^{-x^2}dx$ Over $[0, 0.5]$, e^{-x^2} is concave up because $f''(x) < 0$. So the trapezoid rule's approximation will be greater than the exact value of the integral.

(35) From the hint: $Dist = \int_0^{20} v(t)\,dt$

Using Simpson's rule, $\int_0^{20} v(t)\,dt \approx \frac{20-0}{3 \cdot 4}\left(v(0) + 4v(5) + 2v(10) + 4v(15) + v(20)\right)$
(with $n = 4$)

~~$= \frac{20}{12}(0 + 4 \cdot$~~

$$\int_0^{20} v(t)\,dt \approx \frac{20}{12}\left(0 + 4 \cdot 40\,\frac{mi}{hr} \cdot \frac{\frac{22}{15}\,ft/sec}{mi/hr} + 2 \cdot 60 \cdot \frac{22}{15} + 4 \cdot 75 \cdot \frac{22}{15} + 85 \cdot \frac{22}{15}\right)$$

$$= \frac{10}{6}\left(0 + \frac{704}{3} + 176 + 440 + \frac{374}{3}\right) = 1625\tfrac{5}{9} \text{ feet}$$